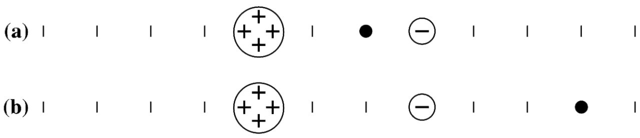
23

THE ELECTRIC FIELD

Conceptual Questions

23.1. A tiny, positive test charge will be placed at the point in space and the force will be measured. From the force measurement and the charge the electric field will be calculated using $E = \dfrac{F}{q}$. The direction of the field will be the same as the direction of the force since q is positive.

23.2.



23.3. $E_3 = E_4 > E_2 > E_1$. The electric field strength is larger in the region where the field lines are closer together $(E_3 \text{ and } E_4)$ and smaller where the field lines are farther apart.

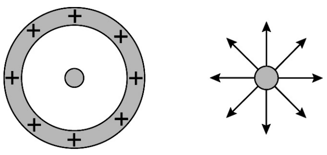
23.4. **(a)** $\dfrac{\lambda_f}{\lambda_i} = \dfrac{(Q_f/L_f)}{(Q_i/L_i)}$. But $Q_i = Q_f$, so $\dfrac{\lambda_f}{\lambda_i} = \dfrac{L_i}{L_f} = 3$

(b) $F \propto \lambda$, so $\dfrac{F_f}{F_i} = 3$

(c) 10 times the original amount of charge would give a constant linear charge density. So the amount of charge to *add* to the original is 9 times the original charge.

23.5. $F = eE = e\left(\dfrac{1}{4\pi E_0}\dfrac{2\lambda}{r}\right)$. If the charge density λ is doubled, then the distance r from the wire must also be doubled for the force to be the same. Thus $r = 2$ cm.

23.6. The electric field at the center is zero. We can think of the straw as being made up of many rings of positive charge. At the center of the ring adding all field vectors gives a resultant electric field equal to zero, as shown in the figure below.



23.7. **(a)** $A_f = \dfrac{A_i}{3.163^2}$, so $\dfrac{\eta_f}{\eta_i} = \dfrac{(Q/A_f)}{(Q/A_i)} = \dfrac{A_i}{A_f} = 3.163^2 = 10$

(b) $\dfrac{F_f}{F_i} = 1$. The total charge on the object does not change as it shrinks. An electron *very far* from the object experiences it as a point charge. By very far, we mean much farther away than the size of the object.

23.8. $N_1 = \dfrac{Q}{A_1} = \dfrac{Q}{\pi r_1^2} = 8\dfrac{\text{nC}}{\text{cm}^2}$, and $N_2 = \dfrac{Q}{A_2} = \dfrac{Q}{\pi r_2^2} = \dfrac{Q}{\pi(2r_1)^2} = \dfrac{Q}{4\pi r_1^2} = \dfrac{1}{4}N_1 = 2\dfrac{\text{nC}}{\text{cm}^2}$.

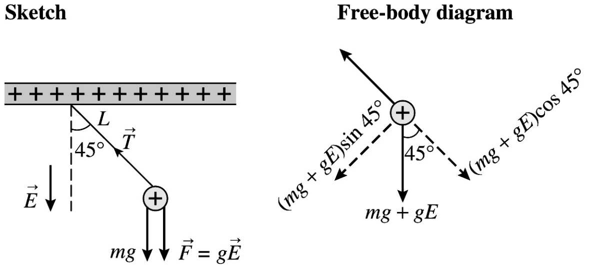
23.9. **(a)** $E = \dfrac{Q}{4\pi E_0 r^2}$. If Q is halved, then E is also halved. Thus $\dfrac{E_f}{E_i} = \dfrac{1}{2}$.

(b) The field outside a sphere is the same as that of a point charge Q located at the center of the sphere. So if the radius of the sphere changes, the field remains the same outside the sphere at the distance $r = 2R$. So $\dfrac{E_f}{E_i} = 1$.

(c) The field outside a uniformly charged sphere is the same as that of a point charge Q located at the center of the sphere. Since r is still greater than R after being halved,

$$E_f = \frac{Q}{4\pi E_0\left(\dfrac{r}{2}\right)^2} = 4E_i \Rightarrow \frac{E_f}{E_i} = 4$$

23.10.



Discharging the ball will cause the restoring force $F = (mg + qE)\sin 45°$ to decrease. Therefore the period of the pendulum will increase. Also, one can think of this as extending the equation for the period of a pendulum by changing the acceleration from simply g to $g + qE/m$. Thus $T = 2\pi\sqrt{\dfrac{L}{g}} \rightarrow 2\pi\sqrt{\dfrac{L}{(g + qE/m)}}$, and as $q \rightarrow 0$, T increases.

23.11. $E_1 = E_2 = E_3 = E_4 = E_5$. The electric field is constant everywhere between the plates. This is indicated by the electric field vectors, which are all the same length and in the same direction.

23.12. **(a)** $\dfrac{E_f}{E_i} = \dfrac{N_f/E_0}{N_i/E_0} = \dfrac{N_f}{N_i} = \dfrac{Q_f/A_f}{Q_i/A_i}$. If Q is doubled (A = constant), $\dfrac{E_f}{E_i} = \dfrac{Q_f}{Q_i} = 2$.

(b) If L is doubled then $A_f = 4A_i$ (Q = constant), so $\dfrac{E_f}{E_i} = \dfrac{A_i}{A_f} = \dfrac{A_i}{4A_i} = \dfrac{1}{4}$.

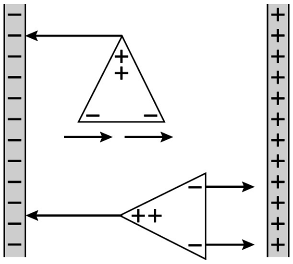
(c) E does not depend on d. $\dfrac{E_f}{E_i} = 1$.

23.13. **(a)** Accelerates to the right.
(b) Remains in place.
(c) Accelerates to the left.

23.14. **(a)** The forces on the two charges are equal, so $\frac{F_p}{F_e} = 1$, $F = qE$, and they each have the same amount of charge and are placed in the same field.

(b) The acceleration of the electron is larger because the electron has smaller mass. Since $a = \frac{F}{m}$, $\frac{a_p}{a_e} = \frac{m_e}{m_p} < 1$.

23.15. **(a)** No, the triangle is not in equilibrium. The electric field is uniform between the plates, so the force on the two positive charges to the left is the same as the total force on the two negative charges to the right. The triangle will rotate counterclockwise as shown.



(b) The triangle will remain in place, since the total force on it is zero.

Exercises and Problems

Exercises

Section 23.1 Electric Field Models

Section 23.2 The Electric Field of Point Charges

23.1. **Model:** The electric field is that of the two charges placed on the y-axis.
Visualize: We denote the upper charge by q_1 and the lower charge by q_2. Because both the charges are positive, their electric fields at P are directed away from the charges.
Solve: The electric field from q_1 is

$$\vec{E}_1 = \left(\frac{1}{4\pi\varepsilon_0} \frac{|q_1|}{r_1^2}, \theta \text{ below } -x\text{-axis} \right) = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(3.0\times10^{-9} \text{ C})}{(0.050 \text{ m})^2 + (0.050 \text{ m})^2} (\cos\theta\hat{i} - \sin\theta\hat{j})$$

Because $\tan\theta = 5 \text{ cm}/5 \text{ cm} = 1$, the angle $\theta = 45°$. Hence,

$$\vec{E}_1 = (5400 \text{ N/C})\left(\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} \right)$$

Similarly, the electric field from q_2 is

$$\vec{E}_2 = \left(\frac{1}{4\pi\varepsilon_0} \frac{|q_2|}{r_2^2}, \theta \text{ above } -x\text{-axis} \right) = (5400 \text{ N/C})\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} \right)$$

$$\Rightarrow \vec{E}_{\text{net at P}} = \vec{E}_1 + \vec{E}_2 = 2(5400 \text{ N/C})\left(\frac{1}{\sqrt{2}} \right)\hat{i} = 7.6\times10^3 \hat{i} \text{ N/C}$$

Thus, the strength of the electric field is 7.6×10^3 N/C and its direction is along the +*x*-axis.

Assess: Because the charges are located symmetrically on either side of the *x*-axis and are of equal value, the *y*-components of their fields will cancel when added.

23.2. Model: The electric field is that due to superposition of the fields of the two charges located on the *y*-axis.

Visualize: We denote the top charge by q_1 and the bottom charge by q_2. The electric fields $(\vec{E}_1$ and $\vec{E}_2)$ of both the positive charges are directed away from their respective charges. With vector addition, they yield the net electric field $\vec{E}_{\text{net}}$ at the point P indicated by the dot.

Solve: The electric fields from q_1 and q_2 are

$$\vec{E}_1=\left(\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2},\text{ along }+x\text{-axis}\right)=\frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(3.0\times10^{-9}\text{ C})}{(0.05\text{ m})^2}\hat{i}=10{,}800\hat{i}\text{ N/C}$$

$$\vec{E}_2=\left(\frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2},\ \theta\text{ above }+x\text{-axis}\right)$$

Because $\tan\theta=10\text{ cm}/5\text{ cm},\theta=\tan^{-1}(2)=63.43°$. So,

$$\vec{E}_2=\frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(6.0\times10^{-9}\text{ C})}{(0.10\text{ m})^2+(0.050\text{ m})^2}(\cos63.43°\,\hat{i}+\sin63.43°\,\hat{j})=(1932\hat{i}+3864\hat{j})\text{ N/C}$$

The net electric field is thus

$$\vec{E}_{\text{net at P}}=\vec{E}_1+\vec{E}_2=(12{,}732\hat{i}+3864\hat{j})\text{ N/C}$$

To find the angle this net vector makes with the *x*-axis, we calculate

$$\tan\phi=\frac{3864\text{ N/C}}{12{,}732\text{ N/C}}\Rightarrow\ \phi=17°$$

Thus, the strength of the electric field at P is

$$E_{\text{net}}=\sqrt{(12{,}732\text{ N/C})^2+(3864\text{ N/C})^2}=13{,}305\text{ N/C}\approx13{,}000\text{ N/C}$$

and $\vec{E}_{\text{net}}$ makes an angle of 17° above the +*x*-axis.

Assess: Because the point P has no special symmetry relative to the charges, we expected the net field to be at an angle relative to the *x*-axis.

23.3. Model: The electric field at the point is found by superposition of the fields due to the two charges located on the *y*-axis.

Visualize: The electric field due to the positive charge q_1 at the point is away from q_1. On the other hand, the electric field due to the negative charge q_2 at the point is toward q_2. These two electric fields are then added vertically to obtain the net electric field at the point.

Solve: The electric field from q_1 is

$$\vec{E}_1=\left(\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2},\theta\text{ below }-x\text{-axis}\right)=\left(\frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(3.0\times10^{-9}\text{ C})}{(0.050\text{ m})^2+(0.050\text{ m})^2}\right)(-\cos\theta\hat{i}-\sin\theta\hat{j})$$

Because $\tan\theta=5\text{ cm}/5\text{ cm},\theta=45°$. So,

$$\vec{E}_1=(5400\text{ N/C})\left(-\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}\right)$$

Similarly, the electric field from q_2 is

$$\vec{E}_2 = \left(\frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2},\ \theta \text{ below } +x\text{-axis}\right) = (5400 \text{ N/C})\left(+\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\Rightarrow \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = 2(5400 \text{ N/C})\left(-\frac{1}{\sqrt{2}}\right)\hat{j} = -7637\hat{j} \text{ N/C} = -7.6\times10^3\ \hat{j} \text{ N/C}$$

Thus, the strength of the electric field is 7.6×10^3 N/C and its direction is vertically downward.

Assess: A quick visualization of the components of the two electric fields shows that the horizontal components cancel.

23.4. Model: The electric field is that due to superposition of the fields of the two charges located on the *y*-axis.

Visualize: We denote the top charge by q_1 and the bottom charge by q_2. The electric field of the positive charge is directed away from q_1 and the electric field of the negative charge is directed toward q_2. With vector addition, they yield the net electric field $\vec{E}_{\text{net}}$ at the point P indicated by the dot.

Solve: The electric fields from q_1 and q_2 are

$$\vec{E}_1 = \left(\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2},\ \text{along } +x\text{-axis}\right) = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(3.0\times10^{-9} \text{ C})}{(0.050 \text{ m})^2}\hat{i} = 10{,}800\hat{i} \text{ N/C}$$

$$\vec{E}_2 = \left(\frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2}, \theta \text{ below } +x\text{-axis}\right)$$

Because $\tan\theta = 10 \text{ cm}/5 \text{ cm}, \theta = \tan^{-1}(2) = 63.43°$. So,

$$\vec{E}_2 = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(6.0\times10^{-9} \text{ C})}{(0.10 \text{ m})^2 + (0.050 \text{ m})^2}(-\cos 63.43°\ \hat{i} - \sin 63.43°\ \hat{j}) = (-1932\hat{i} - 3864\ \hat{j}) \text{ N/C}$$

The net electric field is thus

$$\vec{E}_{\text{net at P}} = \vec{E}_1 + \vec{E}_2 = (8868\hat{i} - 3864\hat{j}) \text{ N/C}$$

To find the angle this net vector makes with the *x*-axis, we calculate

$$\tan\phi = \frac{-3864 \text{ N/C}}{8868 \text{ N/C}} \Rightarrow \phi = -24°$$

Thus, the strength of the electric field at P is

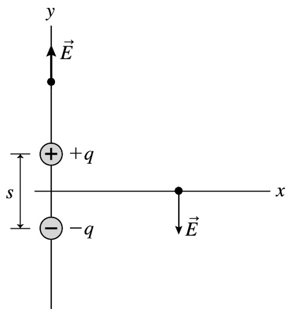
$$E_{\text{net}} = \sqrt{(8868 \text{ N/C})^2 + (3864 \text{ N/C})^2} = 9763 \text{ N/C} \approx 9800 \text{ N/C}$$

and $\vec{E}_{\text{net}}$ makes an angle of 24° below the +*x*-axis.

Assess: Because the point P has no special symmetry relative to the charges, we expected the net field to be at an angle relative to the *x*-axis.

23.5. Model: The distances to the observation points are large compared to the size of the dipole, so model the field as that of a dipole moment.

Visualize:



The dipole consists of charges ±q along the y-axis. The electric field in **(a)** points up. The field in **(b)** points down.
Solve: **(a)** The electric field at (0 cm, 10 cm), which is at $r = 0.10$ m along the axis of the dipole, is

$$\vec{E} = 360\hat{j}\ \text{N/C} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

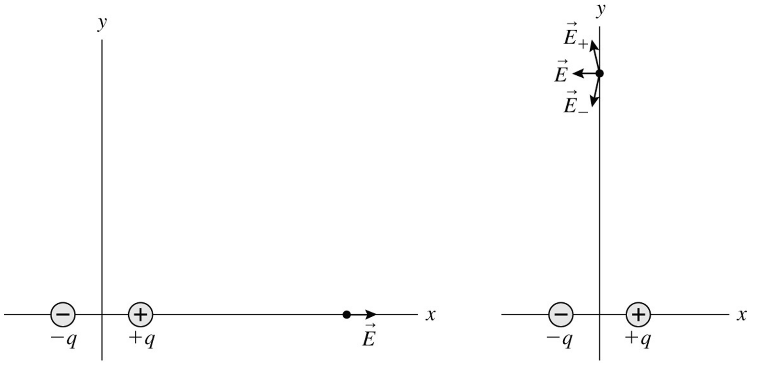
$$\Rightarrow\quad \vec{p} = \frac{r^3\vec{E}}{2(1/4\pi\varepsilon_0)} = \frac{(0.10\ \text{m})^3(360\hat{j}\ \text{N/C})}{2(9.0\times10^9\ \text{N m}^2/\text{C}^2)} = (2.0\times10^{-11}\hat{j}\ \text{C m})$$

By definition, the dipole moment is $\vec{p} = 2.0\times10^{-11}\hat{j}\ \text{C m} = (qs,\ \text{from} - \text{to} +) = q(0.010\ \text{m})\ \hat{j}.$ Thus

$$q = \frac{2.0\times10^{-11}\ \text{C m}}{0.010\ \text{m}} = 2.0\times10^{-9}\ \text{C} = 2.0\ \text{nC}$$

(b) Point (10 cm, 0 cm) is in the plane perpendicular to the dipole. The electric field $\vec{E} = -x(1/4\pi\varepsilon_0)\vec{p}/r^3$ is half the strength of the field at an equal distance r on the axis of the dipole. Hence the field strength at this point is 180 N/C.

23.6. Model: The distances to the observation points are large compared to the size of the dipole, so model the field as that of a dipole moment.
Visualize:



(a) **(b)**

The dipole consists of charges ±q along the x-axis. The electric field in **(a)** points right. The field in **(b)** points left.
Solve: **(a)** The dipole moment is

$$\vec{p} = (qs,\ \text{from} - \text{to} +) = (1.0\times10^{-9}\ \text{C})(0.0020\ \text{m})\hat{i} = 2.0\times10^{-12}\hat{i}\ \text{C m}$$

The electric field at (10 cm, 0 cm), which is at distance $r = 0.10$ m along the axis of the dipole, is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} = (9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{2(2.0\times10^{-12}\hat{i}\ \text{C m})}{(0.10\ \text{m})^3} = 36\hat{i}\ \text{N/C}$$

The field strength, which is all we're asked for, is 36 N/C.
(b) The electric field at (0 cm, 10 cm), which is at $r = 0.10$ m in the plane perpendicular to the electric dipole, is

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} = -(9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{2.0\times10^{-12}\hat{i}\ \text{C m}}{(0.10\ \text{m})^3} = -18.0\hat{i}\ \text{N/C}$$

The field strength at this point is 18 N/C.

23.7. Model: The electret has a dipole field.
Visualize: The ball feels a force $F = qE$ from the dipole field. We are given $r = 0.25$ m, $p = 1.0\times10^{-7}\ \text{C}\cdot\text{m}$, and $q = 25$ nC.

Solve: The force on the ball is its charge multiplied by the electric field (due to the dipole) at the ball.

$$E_{\text{dipole}} = K\frac{2p}{r^3}$$

$$F = qE_{\text{dipole}} = (q)\left(K\frac{2p}{r^3}\right) = (25\text{ nC})\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\,\frac{2(1.0\times10^{-7}\ \text{C}\cdot\text{m})}{(0.25\ \text{m})^3}\right) = 2.9\times10^{-3}\ \text{N}$$

Assess: This small force seems reasonable in this situation.

Section 23.3 The Electric Field of a Continuous Charge Distribution

23.8. Model: We will assume that the wire is thin and that the charge lies on the wire along a *line*.
Solve: From Equation 23.17, the electric field strength of an infinitely long line of charge having linear charge density λ is

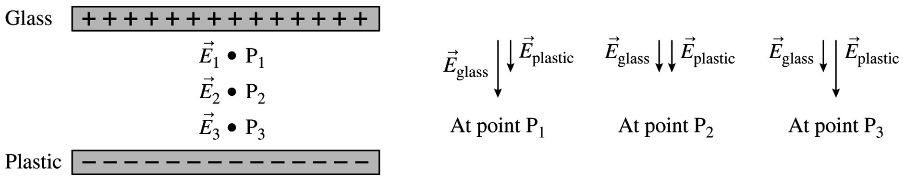
$$E_{\text{line}} = \frac{1}{4\pi\varepsilon_0}\frac{2|\lambda|}{r}$$

$$\Rightarrow E_{\text{line}}(r = 5.0\text{ cm}) = \frac{1}{4\pi\varepsilon_0}\frac{2|\lambda|}{5.0\times10^{-2}\ \text{m}} \qquad E_{\text{line}}(r = 10.0\text{ cm}) = \frac{1}{4\pi\varepsilon_0}\frac{2|\lambda|}{10.0\times10^{-2}\ \text{m}}$$

Dividing the above two equations gives

$$E_{\text{line}}(r = 5.0\text{ cm}) = \left(\frac{10.0\times10^{-2}\ \text{m}}{5.0\times10^{-2}\ \text{m}}\right)E_{\text{line}}(r = 10.0\text{ cm}) = 2(2000\text{ N/C}) = 4.00\times10^3\ \text{N/C} = 4000\text{ N/C}$$

23.9. Model: The rods are thin. Assume that the charge lies along a *line*.
Visualize:



The electric field of the positively charged glass rod points away from the glass rod, whereas the electric field of the negatively charged plastic rod points toward the plastic rod. The electric field strength is the magnitude of the electric field and is always positive.
Solve: Example 23.3 shows that the electric field strength in the plane that bisects a charged rod is

$$E_{\text{rod}} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

The electric field from the glass rod at $r = 1$ cm from the glass rod is

$$E_{\text{glass}} = (9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{10\times10^{-9}\ \text{C}}{(0.01\text{ m})\sqrt{(0.01\text{ m})^2 + (0.05\text{ m})^2}} = 1.765\times10^5\ \text{N/C}$$

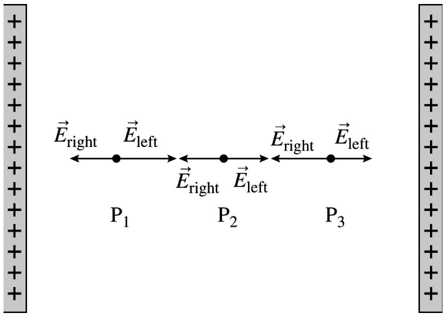
The electric fields from the glass rod at $r = 2$ cm and $r = 3$ cm are 0.835×10^5 N/C and 0.514×10^5 N/C. The electric field from the plastic rod at distances 1 cm, 2 cm, and 3 cm from the plastic rod are the same as for the glass rod. Point P_1 is 1.0 cm from the glass rod and is 3.0 cm from the plastic rod, point P_2 is 2 cm from both rods, and point

P_3 is 3 cm from the glass rod and 1 cm from the plastic rod. Because the direction of the electric fields at P_1 is the same, the net electric field strength 1 cm from the glass rod is the sum of the fields from the glass rod at 1 cm and the plastic rod at 3 cm. Thus

$$\text{At } 1.0\text{ cm}\quad E = 1.765\times10^5\text{ N/C} + 0.514\times10^5\text{ N/C} = 2.3\times10^5\text{ N/C}$$
$$\text{At } 2.0\text{ cm}\quad E = 0.835\times10^5\text{ N/C} + 0.835\times10^5\text{ N/C} = 1.67\times10^5\text{ N/C}$$
$$\text{At } 3.0\text{ cm}\quad E = 0.514\times10^5\text{ N/C} + 1.765\times10^5\text{ N/C} = 2.3\times10^5\text{ N/C}$$

Assess: The electric field strength in the space between the two rods goes through a minimum. This point is exactly in the middle of the line connecting the two rods. Also, note that the arrows shown in the figure are not to scale.

23.10. Model: The rods are thin. Assume that the charge lies along a *line*.
Visualize:



Because both the rods are positively charged, the electric field from each rod points away from the rod. Because the electric fields from the two rods are in opposite directions at P_1, P_2, and P_3, the net field strength at each point is the difference of the field strengths from the two rods.

Solve: Example 23.3 gives the electric field strength in the plane that bisects a charged rod:

$$E_{\text{rod}} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

The electric field from the rod on the right at a distance of 1 cm from the rod is

$$E_{\text{right}} = (9.0\times10^9\text{ N m}^2/\text{C}^2)\frac{10\times10^{-9}\text{ C}}{(0.01\text{ m})\sqrt{(0.01\text{ m})^2 + (0.05\text{ m})^2}} = 1.765\times10^5\text{ N/C}$$

The electric field from the rod on the right at distances 2 cm and 3 cm from the rod are 0.835×10^5 N/C and 0.514×10^5 N/C. The electric fields produced by the rod on the left at the same distances are the same. Point P_1 is 1.0 cm from the rod on the left and is 3.0 cm from the rod on the right. Because the electric fields at P_1 have opposite directions, the net electric field strengths are

$$\text{At } 1.0\text{ cm}\quad E = 1.765\times10^5\text{ N/C} - 0.514\times10^5\text{ N/C} = 1.3\times10^5\text{ N / C}$$
$$\text{At } 2.0\text{ cm}\quad E = 0.835\times10^5\text{ N/C} - 0.835\times10^5\text{ N/C} = 0\text{ N/C}$$
$$\text{At } 3.0\text{ cm}\quad E = 1.765\times10^5\text{ N/C} - 0.514\times10^5\text{ N/C} = 1.3\times10^5\text{ N/C}$$

23.11. Model: Assume the glass bead is a point charge; also assume the bead is in the midplane of the rod so we can use $E_{\text{rod}} = K\dfrac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$.

Visualize: We seek Q. We are given $L = 0.10\text{ m}$.
Solve: Use $F = qE$.

$$F_{\text{on bead}} = q_{\text{bead}} E_{\text{rod}} = q_{\text{bead}} K \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

$$\Rightarrow\ |Q| = \frac{F_{\text{on bead}}}{q_{\text{bead}} K} r\sqrt{r^2 + (L/2)^2} = \frac{840\ \mu\text{N}}{(6.0\times10^{-9}\text{ C})(9.0\times10^{9}\text{ N m}^2/\text{C}^2)}(0.040\text{ m})\sqrt{(0.040\text{ m})^2 + (0.050\text{ m})^2} = 40\text{ nC}$$

The bead is repelled by the rod, which means their charges are the same sign. So the rod has a charge of $+40\text{ nC}$.
Assess: The answer is in the ballpark of the charges we have seen so far.

23.12. Model: Assume that the wire is thin and that the charge lies on the wire along a *line*.
Solve: From Equation 23.17, the electric field for an infinite uniformly charged line is

$$E_{\text{line}} = \frac{1}{4\pi\varepsilon_0}\frac{2|\lambda|}{r}$$

where r is the distance from the line in the plane that bisects the line. Solving for the linear charge density,

$$|\lambda| = \frac{rE_{\text{line}}}{2(1/4\pi\varepsilon_0)} = \frac{(0.050\text{ m})(2000\text{ N/C})}{2(9.0\times10^{9}\text{ Nm}^2/\text{C}^2)} = 5.56\times10^{-9}\text{ C/m}$$

The charge in 1.0 cm is

$$|Q| = L|\lambda| = (1.0\times10^{-2}\text{ m})(5.56\times10^{-9}\text{ C/m}) = 5.56\times10^{-11}\text{ C} = 0.056\text{ nC}$$

Because the electric field is directed toward the line, Q is negative. Thus $Q = -0.056\text{ nC}$.

23.13. Model: For a nonuniform charge density we will need to integrate: $Q = \int dQ$.

Visualize: We are given $\lambda(x) = (2.0\text{ nC/cm})e^{-|x|/(6.0\text{ cm})}$. Put the origin of the coordinate system at the center of the rod.
Solve: Because of the absolute value we will integrate from 0 cm to 6.0 cm and double the result; in that region $|x| = x$.

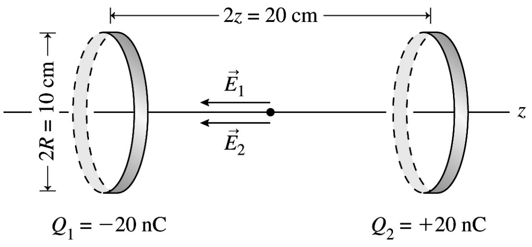
$$Q = \int dQ = 2\int_{0\text{ cm}}^{6.0\text{ cm}} \lambda(x)dx = 2\int_{0\text{ cm}}^{6.0\text{ cm}} (2.0\text{ nC/cm})e^{-x/(6.0\text{ cm})}dx = 2(2.0\text{ nC/cm})\int_{0\text{ cm}}^{6.0\text{ cm}} e^{-x/(6.0\text{ cm})}dx$$

$$= (4.0\text{ nC/cm})\Big[(-6.0\text{ cm})e^{-x/(6.0\text{ cm})}\Big]_{0\text{ cm}}^{6.0\text{ cm}} = (-24.0\text{ nC})\Big[e^{-x/(6.0\text{ cm})}\Big]_{0\text{ cm}}^{6.0\text{ cm}} = (-24.0\text{ nC})\Big[e^{-1} - e^{0}\Big] = 15\text{ nC}$$

Assess: We expected the answer to be positive because the charge density is everywhere positive. The units work out.

Section 23.4 The Electric Fields of Rings, Disks, Planes, and Spheres

23.14. Model: Assume that the rings are thin and that the charge lies along circle of radius R.
Visualize:



The rings are centered on the z-axis.

Solve: **(a)** According to Example 23.4, the field of the left (negative) ring at $z = 10$ cm is

$$(E_1)_z = \frac{zQ_1}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(0.10\ \text{m})(-20\times10^{-9}\ \text{C})}{[(0.10\ \text{m})^2+(0.050\ \text{m})^2]^{3/2}} = -1.288\times10^4\ \text{N/C}$$

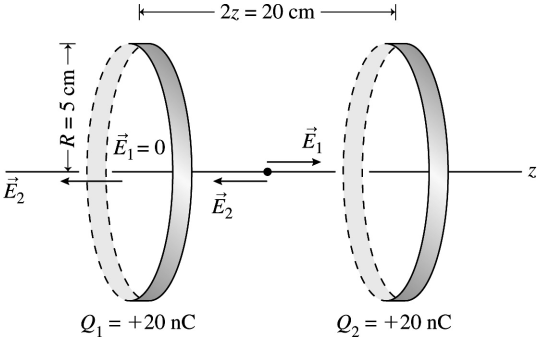
That is, the field is $\vec{E}_1 = (1.288\times10^4\ \text{N/C, left})$. Ring 2 has the same quantity of charge and is at the same distance, so it will produce a field of the same strength. Because Q_2 is positive, $\vec{E}_2$ will also point to the left. The net field at the midpoint is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (2.6\times10^4\ \text{N/C, left})$$

(b) The force is

$$\vec{F} = q\vec{E} = (-1.0\times10^{-9}\ \text{C})(2.6\times10^4\ \text{N/C, left}) = (2.6\times10^{-5}\ \text{N, right})$$

23.15. **Model:** Assume that the rings are thin and that the charge lies along circle of radius R.
Visualize:



Solve: **(a)** Let the rings be centered on the z-axis. According to Example 23.4, the field of the left ring at $z = 10$ cm is

$$(E_1)_z = \frac{zQ_1}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(0.10\ \text{m})(20\times10^{-9}\ \text{C})}{[(0.10\ \text{m})^2+(0.050\ \text{m})^2]^{3/2}} = 1.29\times10^4\ \text{N/C}$$

That is, $\vec{E}_1 = (1.29\times10^4\ \text{N/C, right})$. Ring 2 has the same quantity of charge and is at the same distance, so it will produce a field of the same strength. Because Q_2 is positive, $\vec{E}_2$ will point to the left. The net field at the midpoint between the two rings is $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0\ \text{N/C}$.

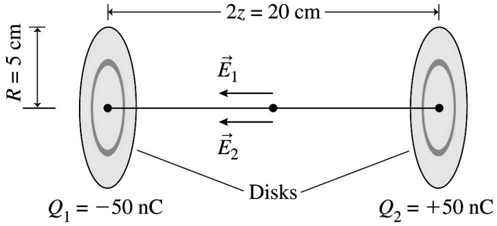
(b) The field of the left ring at $z = 0$ cm is $(E_1)_z = 0\ \text{N/C}$. The field of the right ring at $z = 20$ cm to its left is

$$(E_2)_z = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(0.20\ \text{m})(20\times10^{-9}\ \text{C})}{[(0.20\ \text{m})^2+(0.050)^2]^{3/2}} = 4.1\times10^3\ \text{N/C}$$

$$\Rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2 = 0\ \text{N/C} + (4.1\times10^3\ \text{N/C, left})$$

So the electric field strength is 4.1×10^3 N/C.

23.16. Model: Each disk is a uniformly charged disk. When the disk is charged negatively, the on-axis electric field of the disk points toward the disk. The electric field points away from the disk for a positively charged disk.
Visualize:



Solve: (a) The surface charge density on the disk is

$$|\eta| = \frac{|Q|}{A} = \frac{|Q|}{\pi R^2} = \frac{50\times10^{-9}\text{ C}}{\pi(0.050\text{ m})^2} = 6.366\times10^{-6}\text{ C/m}^2$$

From Equation 23.25, the electric field of the left disk at $z = 0.10$ m is

$$(E_1)_z = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1+R^2/z^2}}\right] = \frac{-6.366\times10^{-6}\text{ C/m}^2}{2(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)}\left[1 - \frac{1}{\sqrt{1+(0.050\text{ m}/0.10\text{ m})^2}}\right] = -38{,}000\text{ N/C}$$

In other words, $\vec{E}_1 = (38{,}000\text{ N/C, left})$. Similarly, the electric field of the right disk at $z = 0.10$ m (to its left) is $\vec{E}_2 = (38{,}000\text{ N/C, left})$. The net field at the midpoint between the two rings is $\vec{E} = \vec{E}_1 + \vec{E}_2 = (7.6\times10^4\text{ N/C, left})$.
(b) The force on the charge is

$$\vec{F} = q\vec{E} = (-1.0\times10^{-9}\text{ C})(7.6\times10^4\text{ N/C, left}) = (7.6\times10^{-5}\text{ N, right})$$

Assess: Note that the force on the negative charge is to the right because the electric field is to the left.

23.17. Model: A spherical shell of charge Q and radius R has an electric field *outside* the sphere that is exactly the same as that of a point charge Q located at the center of the sphere.
Visualize: In the case of a metal ball, the charge resides on its surface. This can then be visualized as a charged spherical shell of radius R.
Solve: Equation 23.30 gives the electric field of a charged spherical shell at a distance $r > R$:

$$\vec{E}_{\text{ball}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

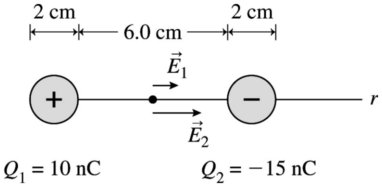
In the present case, $E_{\text{ball}} = 50{,}000$ N/C at $r = \frac{1}{2}(10\text{ cm}) + 2.0\text{ cm} = 7.0\text{ cm} = 0.070\text{ m}$. So,

$$Q = 4\pi\varepsilon_0 r^2 E_{\text{ball}} = \frac{(0.070\text{ m})^2 50{,}000\text{ N/C}}{9.0\times10^9\text{ N m}^2/\text{C}^2} = 2.7\times10^{-8}\text{ C} = 27\text{ nC}$$

23.18. Model: The distance 2.0 mm is very small in comparison to the size of the electrode, so we can model the electrode as a plane of charge.
Solve: From Equation 23.28, the electric field of a plane of charge is

$$E_{\text{plane}} = \frac{\eta}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A} = \frac{80\times10^{-9}\text{ C}}{2(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)(0.20\times0.20\text{ m})} = 1.1\times10^5\text{ N/C}$$

23.19. Model: An insulating sphere of charge Q and radius R has an electric field *outside* the sphere that is exactly the same as that of a point charge Q located at the center of the sphere.
Visualize:



Solve: The electric field of a charged sphere at a distance $r > R$ is given by Equation 23.30:

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

In the present case, $\vec{E} = \vec{E}_1 + \vec{E}_2$, where $\vec{E}_1$ and $\vec{E}_2$ are the fields of the individual spheres. The distance from the center of each sphere to the midpoint between is $r_1 = r_2 = 4$ cm. Thus,

$$E_1 = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(10\times10^{-9}\text{ C})}{(0.040\text{ m})^2} = 5.625\times10^4 \text{ N/C}$$

$$E_2 = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(15\times10^{-9}\text{ C})}{(0.040\text{ m})^2} = 8.438\times10^4 \text{ N/C}$$

The fields point in the same direction, so

$$\vec{E} = (5.625\times10^4 \text{ N/C} + 8.438\times10^4 \text{ N/C, right}) = (1.41\times10^5 \text{ N/C, right}).$$

The electric field will point left when Q_1 and Q_2 are interchanged. The electric field strength in both cases is 1.41×10^5 N/C.

23.20. Model: Assume that the plastic sheets are planes of charge.
Solve: At point 1 the electric fields due to the left sheet and the right sheet are

$$\vec{E}_{\text{left}} = \left(\frac{\eta_0}{2\varepsilon_0}, \text{ toward right}\right) = \frac{\eta_0}{2\varepsilon_0}\hat{\imath} \qquad \vec{E}_{\text{right}} = \left(\frac{3\eta_0}{2\varepsilon_0}, \text{ toward left}\right) = -\frac{3\eta_0}{2\varepsilon_0}\hat{\imath}$$

$$\Rightarrow \vec{E}_{\text{net}} = \vec{E}_{\text{left}} + \vec{E}_{\text{right}} = -\frac{\eta_0}{\varepsilon_0}\hat{\imath}$$

At point 2, $\vec{E}_{\text{left}} = -(\eta_0/2\varepsilon_0)\hat{\imath}, \vec{E}_{\text{right}} = -(3\eta_0/2\varepsilon_0)\hat{\imath}$, and $\vec{E}_{\text{net}} = -(2\eta_0/\varepsilon_0)\hat{\imath}$. At point 3, $\vec{E}_{\text{net}} = +(\eta_0/\varepsilon_0)\hat{\imath}$.

23.21. Model: Assume the carpet is an infinite plane of charge and the dust particle is a point particle. Neglect buoyancy of the air.
Visualize: Use q for the dust particle and Q for the carpet. We are given $Q = -10\ \mu$C. If the particle is suspended at rest then the net force on it is zero. The mass of the dust particle is $m = 2.5\ \mu$g. The gravitational force is downward so the force from the electric field must be upward (positive); since the carpet is negatively charged the dust particle must also be negatively charged.
Solve: For the infinite plane (carpet), $E = \eta/2\varepsilon_0$.

$$\vec{F}_{\text{net}} = q\vec{E}_{\text{carpet}} - mg = q\frac{\eta}{2\varepsilon_0} - mg = q\frac{Q/A}{2\varepsilon_0} - mg = 0 \Rightarrow$$

$$q = mg\frac{2\varepsilon_0 A}{Q} = (2.5\times10^{-9}\text{ kg})(9.8\text{ m/s}^2)\frac{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.0\text{ m}\times4.0\text{ m})}{-10\ \mu\text{C}} = -0.35\text{ pC}$$

Assess: We expected the answer to be negative.

Section 23.5 The Parallel-Plate Capacitor

23.22. Model: The electric field in a region of space between two charged circular disks is uniform.
Solve: The electric field strength inside the capacitor is $E = Q/\varepsilon_0 A$. Thus, the area is

$$A = \frac{Q}{\varepsilon_0 E} = \frac{(3.0\times10^{9})(1.6\times10^{-19}\text{ C})}{(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)(2.0\times10^{5}\text{ N/C})} = 2.71\times10^{-4}\text{ m}^2 = \pi\frac{D^2}{4}$$

$$\Rightarrow D = \sqrt{\frac{4A}{\pi}} = 1.9\text{ cm}$$

Assess: As long as the spacing is much less than the plate dimensions, the electric field is independent of the spacing and depends only on the diameter of the plates.

23.23. Model: The electric field is uniform in a region of space between closely spaced capacitor plates.
Solve: The electric field inside a capacitor is $E = Q/\varepsilon_0 A$. Thus, the charge needed to produce a field of strength E is

$$Q = \varepsilon_0 AE = (8.85\times10^{-12}\text{ C}^2/\text{N m}^2)\pi(0.030\text{ m})^2(1.0\times10^{6}\text{ N/C}) = 25\text{ nC}$$

Thus, one plate has a charge of 25 nC and the other has a charge of −25 nC.
Assess: Note that the capacitor as a whole has no net charge.

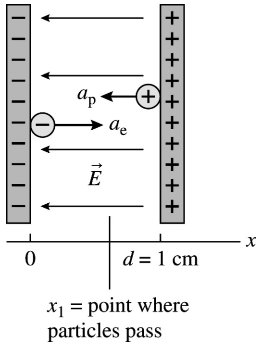
23.24. Model: The electric field in a region of space between the plates of a parallel-plate capacitor is uniform.
Solve: The electric field inside a capacitor is $E = Q/\varepsilon_0 A$. Thus, the charge needed to produce a field of strength E is

$$Q = \varepsilon_0 AE = (8.85\times10^{-12}\text{ C}^2/\text{N m}^2)(0.040\text{ m}\times0.040\text{ m})(3.0\times10^{6}\text{ N/C}) = 42\text{ nC}$$

The number of electrons transferred from one plate to the other is

$$\frac{42\times10^{-9}\text{ C}}{1.60\times10^{-19}\text{ C}} = 2.7\times10^{11}$$

23.25. Model: The parallel plates form a parallel-plate capacitor. The electric field inside a parallel-plate capacitor is a uniform field, so the electron and proton will have constant acceleration.
Visualize:



The negative plate is at $x = 0$ m and the positive plate is at $x = d = 1$ cm.

Solve: Both particles accelerate from rest ($v_0 = 0$ m/s), so at time t their positions are

$$x_e = x_{e0} + \frac{1}{2}a_e t^2 = \frac{1}{2}a_e t^2 \qquad x_p = x_{p0} + \frac{1}{2}a_p t^2 = d + \frac{1}{2}a_p t^2$$

At some instant of time t_1 the electron and proton have the same position: $x_e = x_p = x_1$. This is the point where they pass each other. At this instant,

$$x_1 = \frac{1}{2}a_e t_1^2 \qquad x_1 = d + \frac{1}{2}a_p t_1^2$$

These are two equations in the two unknowns x_1 and t_1. From the first equation, $\frac{1}{2}t_1^2 = x_1/a_e$. Using this in the second equation gives

$$x_1 = d + \frac{a_p}{a_e}x_1 \Rightarrow x_1 = \frac{d}{1 + a_p/a_e}$$

To finish, we need to find the accelerations of the electron and proton. Both particles are in a parallel-plate capacitor with $E_{cap} = Q/\varepsilon_0 A$. The field points to the left, so $E_x = -Q/\varepsilon_0 A$. The proton's acceleration is

$$a_p = \frac{F_p}{m_p} = \frac{q_p E_x}{m_p} = \frac{e(-Q/\varepsilon_0 A)}{m_p} = -\frac{eQ/\varepsilon_0 A}{m_p}$$

The proton's acceleration is negative, as expected. For the electron,

$$a_e = \frac{F_e}{m_e} = \frac{q_e E_x}{m_e} = \frac{-e(-Q/\varepsilon_0 A)}{m_e} = \frac{eQ/\varepsilon_0 A}{m_e}$$

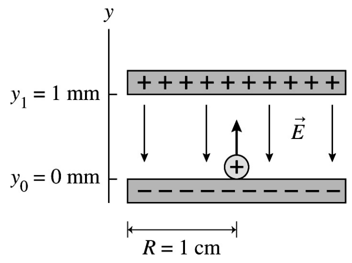
Consequently, the acceleration ratio is $a_p/a_e = m_e/m_p$. Using this, the point where the two charges pass is

$$x_1 = \frac{d}{1 + m_e/m_p} = \frac{1\,\text{cm}}{1 + 9.11\times10^{-31}\,\text{kg}/1.67\times10^{-27}\,\text{kg}} = 0.9995\,\text{cm}$$

Assess: This is very close to where the proton starts. Since there is a factor of ~1800 difference in the masses of the proton and electron, the electron accelerates much faster than the proton.

Section 23.6 Motion of a Charged Particle in an Electric Field

23.26. Model: The disks form a parallel-plate capacitor. The electric field inside a parallel-plate capacitor is a uniform field, so the proton will have a constant acceleration.
Visualize:



Solve: (a) The two disks form a parallel-plate capacitor with surface charge density

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{10\times10^{-9}\,\text{C}}{\pi(0.010\,\text{m})^2} = 3.18\times10^{-5}\,\text{C/m}^2$$

From Equation 23.31 and the equation for η above, the field strength inside a capacitor is

$$E = \frac{\eta}{\varepsilon_0} = \frac{3.18\times10^{-5}\,\text{C/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N m}^2} = 3.6\times10^{6}\,\text{N/C}$$

(b) The electric field points toward the negative plate, so in the coordinate system of the figure $\vec{E} = -3.6\times10^6\,\hat{j}$ N/C. The field exerts forces $\vec{F} = q_{\text{proton}}\vec{E} = e\vec{E}$ on the proton, causing an acceleration with a y-component that is

$$a_y = \frac{F}{m} = \frac{eE_y}{m} = \frac{(1.6\times10^{-19}\text{ C})(-3.6\times10^6\text{ N/C})}{1.67\times10^{-27}\text{ kg}} = -3.45\times10^{14}\text{ m/s}^2$$

After the proton is launched, this acceleration will cause it to lose speed. To just barely reach the positive plate, it should reach $v_1 = 0$ m/s at $y_1 = 1$ mm. The kinematic equation of motion is

$$v_1^2 = 0\text{ m}^2/\text{s}^2 = v_0^2 + 2a_y\Delta y$$

$$\Rightarrow v_0 = \sqrt{-2a_y\Delta y} = \sqrt{-2(-3.45\times10^{14}\text{ m/s}^2)(0.0010\text{ m})} = 8.3\times10^5\text{ m/s}$$

Assess: The acceleration of the proton in the electric field is enormous in comparison to the gravitation acceleration g. That is why we did not explicitly consider g in our calculations.

23.27. Model: Model the bee as a point particle. Neglect buoyancy of the air.

Visualize: We are given $\vec{E} = (100\text{ N/C, down})$, $q = 23$ pC and $m = 100\ \mu$g.

Solve: (a)

$$\frac{F_\text{e}}{F_\text{g}} = \frac{qE}{mg} = \frac{(23\text{ pC})(100\text{ N/C})}{(100\times10^{-9}\text{ kg})(9.8\text{ m/s}^2)} = 0.0023$$

(b) Since the bee is positively charged we need the electric field to be upward so the force of the electric field will also be upward.

$$F_\text{net} = qE - mg = 0 \Rightarrow E = \frac{mg}{q} = \frac{(100\times10^{-9}\text{ kg})(9.8\text{ m/s}^2)}{23\text{ pC}} = 43\text{ kN/C, up}$$

Assess: It would take an extraordinary field to allow the bee to hang suspended; so the bee uses the air and beats its wings.

23.28. Model: A uniform electric field causes a charge to undergo constant acceleration.

Solve: Kinematics yields the acceleration of the electron.

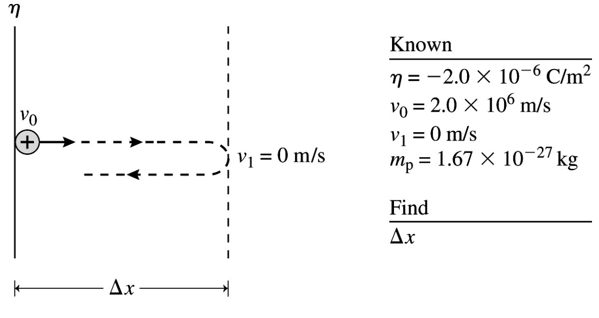
$$v_1{}^2 = v_0{}^2 + 2a\Delta x \Rightarrow a = \frac{v_1{}^2 - v_0{}^2}{2\Delta x} = \frac{(4.0\times10^7\text{ m/s})^2 - (2.0\times10^7\text{ m/s})^2}{2(0.012\text{ m})} = 5.0\times10^{16}\text{ m/s}^2$$

The magnitude of the electric field required to obtain this acceleration is

$$E = \frac{F_\text{net}}{e} = \frac{m_e a}{e} = \frac{(9.11\times10^{-31}\text{ kg})(5.0\times10^{16}\text{ m/s}^2)}{1.6\times10^{-19}\text{ C}} = 2.8\times10^5\text{ N/C.}$$

23.29. Model: The infinite negatively charged plane produces a uniform electric field that is directed toward the plane.

Visualize:


η
v_0
+
$v_1 = 0$ m/s
Δx
Known
$\eta = -2.0\times10^{-6}$ C/m^2
$v_0 = 2.0\times10^6$ m/s
$v_1 = 0$ m/s
$m_\text{p} = 1.67\times10^{-27}$ kg
Find
Δx

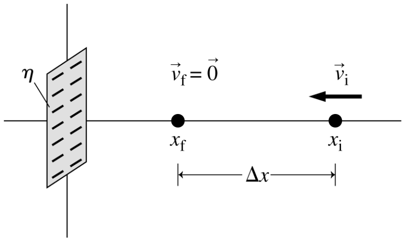
Solve: From the kinematic equation of motion $v_1^2 = 0 = v_0^2 + 2a\Delta x$ and $F = qE = ma$,

$$a = \frac{qE}{m} = \frac{-v_0^2}{2\Delta x} \Rightarrow \Delta x = \frac{-mv_0^2}{2qE}$$

Furthermore, the electric field of a plane of charge with surface charge density η is $E = \eta/2\varepsilon_0$. Thus,

$$\Delta x = \frac{-mv_0^2\varepsilon_0}{q\eta} = \frac{-(1.67\times10^{-27}\text{ kg})(2.0\times10^6\text{ m/s})^2(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)}{(1.60\times10^{-19}\text{ C})(-2.0\times10^{-6}\text{ C/m}^2)} = 0.18\text{ m}$$

23.30. Model: Model the plate as infinite so the electric field is uniform (constant).
Visualize: The plate must be negatively charged since it repels the electron.



Known
$x_i = 0.75$ m
$x_f = 0.15$ m
$v_i = 8300$ km/s
$v_f = 0$

Find
η

The force on the particle is constant, so the acceleration is constant so we can use the kinematic equations.
Solve: Use the kinematic equations to find the acceleration. With the final speed as zero we have

$$v_f^2 = v_i^2 + 2a\Delta x \Rightarrow a = \frac{-v_i^2}{2\Delta x}$$

Now apply Newton's second law.

$$F_{\text{net}} = qE = q\frac{\eta}{2\varepsilon_0} = ma \Rightarrow$$

$$\eta = \frac{2\varepsilon_0 ma}{q} = \frac{2\varepsilon_0 m\left(\dfrac{-v_i^2}{2\Delta x}\right)}{q} = \frac{-v_i^2\varepsilon_0 m}{q\Delta x} = \frac{-v_i^2\varepsilon_0 m_e}{(-e)\Delta x}$$

$$= \frac{-(8300\text{ km/s})^2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(9.11\times10^{-31}\text{ kg})}{(-1.6\times10^{-19}\text{ C})(-0.60\text{ m})} = -5.8\text{ nC/m}^2$$

Assess: This seems to be in the ballpark of other surface charge densities we've seen.

23.31. Model: Assume the oil drop is a point charge. Model the plane as infinite. The negative drop will be attracted to the positive plane. Ignore gravity.
Solve: We will combine a number of equations together. Solve the equation for the field due to an infinite plane of charge for the surface charge density: $\eta = 2\varepsilon_0 E$. The only force is the electric force, so it is the net force: $F = qE = ma \Rightarrow E = \dfrac{ma}{q_{\text{drop}}}$. Also solve $v_f^2 - v_i^2 = 2a\Delta x$ for a where $v_i = 0$: $a = \dfrac{v_f^2}{2\Delta x}$. We also need the mass of the drop: $m = \rho V = \rho\frac{4}{3}\pi R^3$. Combine all these together.

$$\eta = 2\varepsilon_0 E = 2\varepsilon_0\frac{ma}{q_{\text{drop}}} = 2\varepsilon_0\frac{m}{q_{\text{drop}}}\frac{v_f^2}{2\Delta x} = \varepsilon_0\frac{m}{q_{\text{drop}}}\frac{v_f^2}{\Delta x} = \varepsilon_0\frac{\rho\frac{4}{3}\pi R^3}{q_{\text{drop}}}\frac{v_f^2}{\Delta x}$$

$$= (8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)\frac{(900\text{ kg/m}^3)\frac{4}{3}\pi(0.50\ \mu\text{m})^3}{25(1.60\times10^{-19}\text{ C})}\frac{(3.5\text{ m/s})^2}{0.0020\text{ m}} = 6.4\ \mu\text{C/m}^2$$

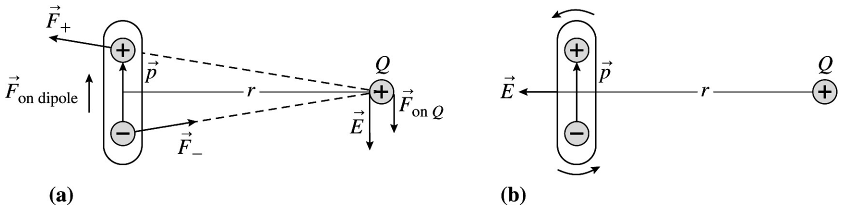
Assess: The answer is in the ballpark of the surface charge density in Example 23.6. Careful analysis will confirm that the units work out correctly.

Section 23.7 Motion of a Dipole in an Electric Field

23.32. Model: The external electric field exerts a torque on the dipole moment of the water molecule.
Solve: From Equation 23.36, the torque exerted on a dipole moment by an electric field is $\tau = pE\sin\theta$. The maximum torque is exerted when $\sin\theta = 1$ or $\theta = 90°$. Thus,

$$\tau_{\text{max}} = pE = (6.2\times10^{-30}\text{ C m})(5.0\times10^{8}\text{ N/C}) = 3.1\times10^{-21}\text{ N m}$$

23.33. Model: Because $r >> s$, we can use $\vec{E}_{\text{dipole}} = -K\,\vec{p}/r^3$ for the electric field in the plane bisecting the dipole.
Visualize:



Solve: (a) From Newton's third law, the force of Q on the dipole is equal and opposite to the force of the dipole on Q. You can see from the diagram that $\vec{F}_{\text{dipole on }Q}$ is down and $\vec{F}_{Q\text{ on dipole}}$ is up, in the direction of $\vec{p}$. The magnitude of the dipole field at the position of Q is

$$E_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{qs}{r^3}$$

The magnitude of $\vec{F}_{\text{dipole on }Q}$ is

$$F_{\text{dipole on }Q} = QE_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0}\frac{qQs}{r^3}$$

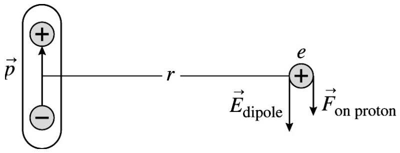
$\vec{F}_{Q\text{ on dipole}}$ has the same magnitude.

(b) The electric field of charge Q exerts a torque on the dipole. The field $\vec{E}$ is perpendicular to the dipole $\vec{p}$, so $\theta = 90°$. The torque is given by $\vec{\tau} = \vec{p}\times\vec{E}$.

$$\tau = pE\sin\theta = (qs)\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)\sin 90° = \frac{1}{4\pi\varepsilon_0}\frac{qQs}{r^2}$$

Assess: Note the similar form, but the torque has different dimensions $(\text{N}\cdot\text{m})$, hence the different power of r in the denominator.

23.34. Model: The size of a molecule is ≈ 0.1 nm. The proton is 2.0 nm away, so $r >> s$ and we can use Equation 23.11 for the electric field in the plane that bisects the dipole.
Visualize:



Solve: You can see from the diagram that $\vec{F}_{\text{dipole on proton}}$ is opposite to the direction of $\vec{p}$. The magnitude of the dipole field at the position of the proton is

$$E_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = (9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{5.0\times10^{-30}\ \text{C m}}{(2.0\times10^{-9}\ \text{m})^3} = 5.624\times10^5\ \text{N/C}$$

The magnitude of $\vec{F}_{\text{dipole on proton}}$ is

$$F_{\text{dipole on proton}} = eE_{\text{dipole}} = (1.60\times10^{-19}\ \text{C})(5.624\times10^5\ \text{N/C}) = 9.0\times10^{-13}\ \text{N}$$

Including the direction, the force is $\vec{F}_{\text{dipole on proton}} = (9.0\times10^{-13}\ \text{N, direction opposite } \vec{p})$.

Problems

23.35. Model: The electric field is that of three point charges q_1, q_2, and q_3. Assume the charges are in the *x*-*y* plane.

Visualize: The 5.0 nC charge is q_1, the −5.0 nC charge is q_3, and the 10 nC charge is q_2. The net electric field at the dot is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. The procedure will be to find the magnitudes of the electric fields, to write them in component form, and to add the components.

Solve: **(a)** The electric field produced by q_1 is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(5.0\times10^{-9}\ \text{C})}{(0.020\ \text{m})^2} = 112{,}500\ \text{N/C}$$

$\vec{E}_1$ points away from q_1, so in component form $\vec{E}_1 = 112{,}500\hat{i}$ N/C.

The electric field produced by q_2 is $E_2 = 56{,}250$ N/C. $\vec{E}_2$ points away from q_2, so $\vec{E}_2 = -56{,}250\hat{j}$ N/C.

Finally, the electric field produced by q_3 is

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{|q_3|}{r_3^2} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(5.0\times10^{-9}\ \text{C})}{(0.020\ \text{m})^2 + (0.040\ \text{m})^2} = 22{,}500\ \text{N/C}$$

$\vec{E}_3$ points toward q_3 and makes an angle $\phi = \tan^{-1}(4/2) = 63.43°$ with the *x*-axis. So,

$$\vec{E}_3 = -E_3\cos\phi\hat{i} + E_3\sin\phi\hat{j} = (-10064\hat{i} + 20{,}124\hat{j})\ \text{N/C}$$

Adding these three vectors gives

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = (102{,}436\hat{i} - 36{,}126\hat{j})\ \text{N/C} = (1.0\times10^5\hat{i} - 3.6\times10^4\hat{j})\ \text{N/C}$$

This is in component form.

(b) The magnitude of the field is

$$E_{\text{net}} = \sqrt{E_x^2 + E_y^2} = \sqrt{(102{,}436\ \text{N/C})^2 + (-36{,}126\ \text{N/C})^2} = 108{,}619\ \text{N/C} = 1.1\times10^5\ \text{N/C}$$

and its angle from the *x*-axis is $\theta = \tan^{-1}\left(\left|E_y/E_x\right|\right) = -19.4°$. We can also write $\vec{E}_{\text{net}} = (1.1\times10^5\ \text{N/C}, 19.4°\ \text{CW from the } +x\text{-axis})$.

23.36. Model: The electric field is that of three point charges q_1, q_2, and q_3. Assume the charges are in the *x*-*y* plane.

Visualize: The −5.0 nC charge is q_1, the bottom 10 nC charge is q_2, and the top 10 nC charge is q_3. The net electric field at the dot is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. The procedure will be to find the magnitudes of the electric fields, to write them in component form, and to add the components.

Solve: **(a)** The electric field produced by q_1 is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(5.0\times10^{-9}\text{ C})}{(0.020\text{ m})^2} = 112{,}500 \text{ N/C}$$

$\vec{E}_1$ points toward q_1, so in component form $\vec{E}_1 = 112{,}500\hat{j}$ N/C.

The electric field produced by q_2 is $E_2 = 56{,}250$ N/C. $\vec{E}_2$ points away from q_2, so $\vec{E}_2 = -56{,}250\hat{i}$ N/C.
Finally, the electric field produced by q_3 is

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{|q_3|}{r_3^2} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(10\times10^{-9}\text{ C})}{(0.020\text{ m})^2 + (0.040\text{ m})^2} = 45{,}000 \text{ N/C}$$

$\vec{E}_3$ points away from q_3 and makes an angle $\phi = \tan^{-1}(2/4) = 26.6°$ with the x-axis. So,

$$\vec{E}_3 = -E_3\cos\phi\hat{i} - E_3\sin\phi\hat{j} = (-40{,}250\hat{i} - 20{,}130\hat{j}) \text{ N/C}$$

Adding these three vectors gives

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = (-96{,}500\hat{i} + 92{,}400\hat{j}) \text{ N/C} = (-9.7\times10^4\hat{i} + 9.2\times10^4\hat{j}) \text{ N/C}$$

This is in component form.
(b) The magnitude of the field is

$$E_{\text{net}} = \sqrt{E_x^2 + E_y^2} = \sqrt{(96{,}500 \text{ N/C})^2 + (92{,}400 \text{ N/C})^2} = 133{,}600 \text{ N/C} = 1.34\times10^5 \text{ N/C}$$

and its angle below the $-x$-axis is $\theta = \tan^{-1}\left(\left|E_y/E_x\right|\right) = 44°$. We can also write $\vec{E}_{\text{net}} = (1.34\times10^5 \text{ N/C}, 136° \text{ CCW}$ from the $+x$-axis).

23.37. Model: The electric field is that of three point charges q_1, q_2, and q_3. Assume the charges are in the x-y plane.
Visualize: The -10 nC charge is q_1, the 10 nC charge is q_3, and the -5.0 nC charge is q_2. The net electric field at the dot is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. The procedure will be to find the magnitudes of the electric fields, to write them in component form, and to add the components.
Solve: **(a)** The electric field produced by q_1 is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(10\times10^{-9}\text{ C})}{(0.030\text{ m})^2} = 100{,}000 \text{ N/C}$$

$\vec{E}_1$ points toward q_1, so in component form $\vec{E}_1 = 100{,}000\hat{j}$ N/C.

The electric field produced by q_2 is $E_2 = 18{,}000$ N/C. $\vec{E}_2$ points toward q_2, so $\vec{E}_2 = 18{,}000\hat{i}$ N/C.
Finally, the electric field produced by q_3 is

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{|q_3|}{r_3^2} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(10\times10^{-9}\text{ C})}{(0.030\text{ m})^2 + (0.050\text{ m})^2} = 26{,}471 \text{ N/C}$$

$\vec{E}_3$ points away from q_3 and makes an angle $\phi = \tan^{-1}(3/5) = 31°$ with the x-axis. So,

$$\vec{E}_3 = -E_3\cos\phi\hat{i} - E_3\sin\phi\hat{j} = (-22{,}700\hat{i} - 13{,}634\hat{j}) \text{ N/C}$$

Adding these three vectors gives

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = (-4700\hat{i} + 86{,}366\hat{j}) \text{ N/C} = (-4.7\times10^3\hat{i} + 8.6\times10^4\hat{j}) \text{ N/C}$$

This is in component form.

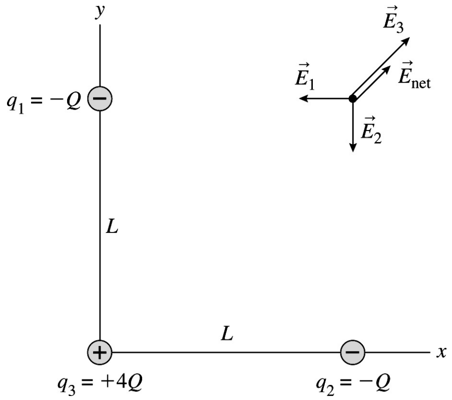
(b) The magnitude of the field is

$$E_{\text{net}} = \sqrt{E_x^2 + E_y^2} = \sqrt{(-4{,}700\ \text{N/C})^2 + (86{,}366\ \text{N/C})^2} = 86{,}494\ \text{N/C} = 8.6\times10^4\ \text{N/C}$$

and its angle from the x-axis is $\theta = \tan^{-1}\left(\left|E_y/E_x\right|\right) = -87°$, but we see that we need to be in the second quadrant so we add 180 °. We can also write $\vec{E}_{\text{net}} = (8.6\times10^4\ \text{N/C}, 93°\ \text{CCW from the} +x\text{-axis})$.

23.38. Model: The electric field is that of three point charges $q_1 = -Q$, $q_2 = -Q$, and $q_3 = +4Q$.

Visualize: Assume the charges are in the x-y plane. The net electric field at point P is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. The procedure will be to find the magnitudes of the electric fields, to write them in component form, and to add the components.



Solve: The electric field produced by q_1 points toward q_1 and is given by

$$\vec{E}_1 = -\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}\right)\hat{i}$$

The electric field produced by q_2 points toward q_2 and is given by

$$\vec{E}_2 = -\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}\right)\hat{j}$$

The electric field produced by q_3 is

$$E_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4Q}{L^2+L^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{L^2}\right)$$

$\vec{E}_3$ points away from q_3 and makes an angle $\phi = \tan^{-1}(L/L) = 45°$ with the x-axis. So

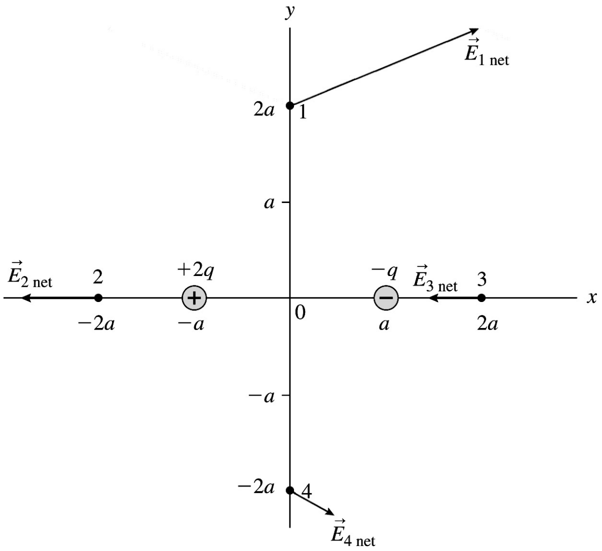
$$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{L^2}\right)(\cos\phi\hat{i} + \sin\phi\hat{j}) = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{L^2}\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

Adding these three vectors gives

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}\left[\left(\frac{2}{\sqrt{2}}-1\right)\hat{i} + \left(\frac{2}{\sqrt{2}}-1\right)\hat{j}\right] = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}(\sqrt{2}-1)(\hat{i}+\hat{j})$$

23.39. Model: The electric field is that of two charges $-q$ and $+2q$ located at $x=\pm a$.

Visualize:



Solve: At point 1, the electric field from $-q$ is

$$E_{-q}=\frac{1}{4\pi\varepsilon_0}\frac{|-q|}{(a)^2+(2a)^2}=\frac{1}{4\pi\varepsilon_0}\frac{q}{5a^2}$$

$\vec{E}_{-q}$ points toward $-q$ and makes an angle $\phi_1=\tan^{-1}(2a/a)=63.43°$ below the $-x$-axis, hence

$$\vec{E}_{-q}=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5a^2}\right)(\cos\phi_1\hat{i}-\sin\phi_1\hat{j})=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5a^2}\right)\left(\frac{1}{\sqrt{5}}\hat{i}-\frac{2}{\sqrt{5}}\hat{j}\right)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)(\hat{i}-2\hat{j})$$

The electric field from the $+2q$ is

$$E_{+2q}=\frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2+(2a)^2}=\frac{1}{4\pi\varepsilon_0}\frac{2q}{5a^2}$$

$\vec{E}_{+2q}$ points away from $+2q$ and makes an angle $\phi_1=\tan^{-1}(2a/a)=63.43°$ above the $-x$-axis. So,

$$\vec{E}_{+2q}=\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{5a^2}\right)(\cos\phi_2\hat{i}+\sin\phi_2\hat{j})=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)(2\hat{i}+4\hat{j})$$

Adding these two vectors,

$$\vec{E}_{1\text{ net}}=\vec{E}_{-q}+\vec{E}_{+2q}=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)(3\hat{i}+2\hat{j})$$

At point 2, the electric field from $-q$ points toward $-q$, so

$$\vec{E}_{-q}=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{9a^2}\right)\hat{i}$$

The electric field from $+2q$ points away from $+2q$, so

$$\vec{E}_{+2q}=-\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{a^2}\right)\hat{i}$$

Adding these two vectors,

$$\vec{E}_{2\text{ net}}=\vec{E}_{-q}+\vec{E}_{+2q}=-\frac{1}{4\pi\varepsilon_0}\left(\frac{17q}{9a^2}\right)\hat{i}$$

At point 3, the electric field from $-q$ points toward $-q$, so

$$\vec{E}_{-q} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a^2}\right)\hat{i}$$

The electric field from $+2q$ points away from $+2q$, so

$$\vec{E}_{+2q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{9a^2}\right)\hat{i}$$

Adding these vectors,

$$\vec{E}_{3\text{ net}} = \vec{E}_{-q} + \vec{E}_{+2q} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{7q}{9a^2}\right)\hat{i}$$

Point 4 is a mirror image of point 1. Since $\vec{E}_{1\text{ net}}$ points to the left and up, $\vec{E}_{4\text{ net}}$ has a reversed y-component and points to the left and down. Thus,

$$\vec{E}_{4\text{ net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)(3\hat{i} - 2\hat{j})$$

23.40. Model: The electric field of a dipole is that of two opposite charges $\pm q$ that make the dipole.
Visualize: Please refer to Figure 23.5. The figure shows a dipole aligned on the y-axis, so the x-axis is the bisecting axis. The field at a point on the x-axis is $\vec{E}_{\text{dipole}} = \vec{E}_+ + \vec{E}_-$.
Solve: From the symmetry of the situation we can see that the x-components of the two contributions to the electric field will cancel, that they have equal y-components, and that $\vec{E}_{\text{dipole}}$ points in the $-y$-direction. Thus,

$$(E_{\text{dipole}})_x = 0\text{ N/C} \qquad (E_{\text{dipole}})_y = -2E_+\sin\theta$$

where θ is the angle $\vec{E}_+$ makes with the x-axis. From the geometry of the figure,

$$\sin\theta = \frac{s/2}{[x^2 + (s/2)^2]^{1/2}}$$

For a point charge $+q$, the field is

$$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2 + (s/2)^2}$$

Combining these pieces gives the dipole field at distance x along the bisecting axis:

$$\vec{E}_{\text{dipole}} = -(2)\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2 + (s/2)^2}\frac{s/2}{[x^2 + (s/2)^2]^{1/2}}\hat{j} = -\frac{qs}{4\pi\varepsilon_0(x^2 + s^2/4)^{3/2}}\hat{j}$$

If $x >> s$, then $(x^2 + s^2/4)^{3/2} \approx x^3$. Thus

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\varepsilon_0}\frac{qs\hat{j}}{x^3}$$

If we note that $\vec{p} = qs\hat{j}$ and if we replace x with a more general variable r to denote the distance from the dipole, then

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$$

This is Equation 23.11.

23.41. Model: The electric field is that of two infinite lines of charge extending out of the page.
Visualize: The line charges lie on the x-axis.
Solve: From Equation 23.17, the electric field strength due to an infinite line of charge at a distance r from the line charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2|\lambda|}{r}$$

For the left and right line charges, the electric fields are

$$E_{\text{left}} = E_{\text{right}} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{\sqrt{y^2+(d/2)^2}} = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda}{\sqrt{4y^2+d^2}}$$

$\vec{E}_{\text{left}}$ makes an angle ϕ above the $+x$-axis and $\vec{E}_{\text{right}}$ makes the same angle ϕ above the $-x$-axis. From the geometry of the figure,

$$\cos\phi = \frac{d/2}{\sqrt{y^2+d^2/4}} = \frac{d}{\sqrt{4y^2+d^2}} \qquad \sin\phi = \frac{2y}{\sqrt{4y^2+d^2}}$$

$$\Rightarrow \vec{E}_{\text{left}} = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda}{\sqrt{4y^2+d^2}}\left(\frac{d}{\sqrt{4y^2+d^2}}\hat{i} + \frac{2y}{\sqrt{4y^2+d^2}}\hat{j}\right)$$

$$\vec{E}_{\text{right}} = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda}{\sqrt{4y^2+d^2}}\left(-\frac{d}{\sqrt{4y^2+d^2}}\hat{i} + \frac{2y}{\sqrt{4y^2+d^2}}\hat{j}\right)$$

We now add these two vectors to find $\vec{E}_{\text{net}} = \vec{E}_{\text{left}} + \vec{E}_{\text{right}}$. The x-components cancel to give

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4\lambda}{\sqrt{4y^2+d^2}}\right)(2)\left(\frac{2y}{\sqrt{4y^2+d^2}}\right)\hat{j} = \frac{1}{4\pi\varepsilon_0}\frac{16\lambda y}{(4y^2+d^2)}\hat{j}$$

Thus the field strength is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{16\lambda y}{4y^2+d^2}$$

23.42. Model: The electric field is that of two infinite lines of charge extending out of the page.
Visualize: The line charges lie on the x-axis.
Solve: (a) From Equation 23.17, the electric field strength due to an infinite line of charge at a distance r from the line charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2|\lambda|}{r}$$

For the left and right line charges, the electric fields are

$$E_{\text{left}} = E_{\text{right}} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{\sqrt{y^2+(d/2)^2}} = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda}{\sqrt{4y^2+d^2}}$$

$$\cos\phi = \frac{d/2}{\sqrt{y^2+d^2/4}} = \frac{d}{\sqrt{4y^2+d^2}} \qquad \sin\phi = \frac{2y}{\sqrt{4y^2+d^2}}$$

$$\Rightarrow \vec{E}_{\text{left}} = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda}{\sqrt{4y^2+d^2}}\left(\frac{d}{\sqrt{4y^2+d^2}}\hat{i} + \frac{2y}{\sqrt{4y^2+d^2}}\hat{j}\right)$$

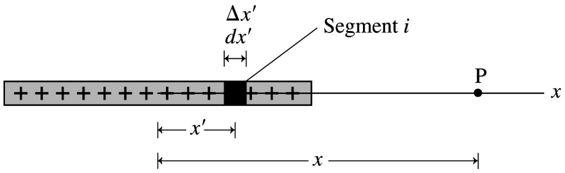
$$\vec{E}_{\text{right}} = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda}{\sqrt{4y^2+d^2}}\left(\frac{d}{\sqrt{4y^2+d^2}}\hat{i} - \frac{2y}{\sqrt{4y^2+d^2}}\hat{j}\right)$$

$$\Rightarrow \vec{E}_{\text{net}} = \vec{E}_{\text{left}} + \vec{E}_{\text{right}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4\lambda}{\sqrt{4y^2+d^2}}\right)(2)\left(\frac{d}{\sqrt{4y^2+d^2}}\right)\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{8\lambda d}{(4y^2+d^2)}\hat{i}$$

Thus the field strength is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{8\lambda d}{4y^2 + d^2}$$

23.43. Model: The electric field is that of a line of charge of length L.
Visualize:



The origin of the coordinate system is at the center of the rod. Divide the rod into many small segments of charge Δq and length $\Delta x'$.

Solve: (a) Segment i creates a small electric field $\vec{E}_i$ at point P that points to the right. The net field $\vec{E}$ will point to the right and have $E_y = E_z = 0$ N/C. The distance to segment i is x', so

$$E_\mathrm{i} = (E_i)_x = \frac{\Delta q}{4\pi\varepsilon_0 (x - x_i')^2} \Rightarrow E_x = \sum (E_i)_x = \frac{1}{4\pi\varepsilon_0}\sum \frac{\Delta q}{(x - x_i')^2}$$

Δq is not a coordinate, so before converting the sum to an integral we must relate the charge Δq to length $\Delta x'$. This is done through the linear charge density $\lambda = Q/L$, from which

$$\Delta q = \lambda \Delta x' = \frac{Q}{L}\Delta x'$$

With this charge, the sum becomes

$$E_x = \frac{(Q/L)}{4\pi\varepsilon_0}\sum_i \frac{\Delta x'}{(x - x_i')^2}$$

Now we let $\Delta x' \to dx'$ and replace the sum by an integral from $x' = -\frac{1}{2}L$ to $x' = +\frac{1}{2}L$. Thus,

$$E_x = \frac{(Q/L)}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} \frac{dx'}{(x - x')^2} = \frac{(Q/L)}{4\pi\varepsilon_0}\left[\frac{1}{x - x'}\right]_{-L/2}^{L/2} = \frac{(Q/L)}{4\pi\varepsilon_0}\frac{L}{x^2 - L^2/4} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2 - L^2/4}\hat{i}$$

The electric field strength at x is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2 - L^2/4}$$

(b) For $x >> L$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$$

That is, the line charge behaves like a point charge.
(c) Substituting into the above formula

$$E = (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{3.0\times10^{-9}\text{ C}}{(3.0\times10^{-2}\text{ m})^2 - \left(\frac{1}{2}\times5.0\times10^{-2}\text{ m}\right)^2} = 9.8\times10^4 \text{ N/C}$$

23.44. Model: The electric field is that of a line charge of length L.
Visualize: Let the bottom end of the rod be the origin of the coordinate system. Divide the rod into many small segments of charge Δq and length $\Delta y'$. Segment i creates a small electric field at the point P that makes an angle θ with the horizontal. The field has both x and y components, but $E_z = 0$ N/C. The distance to segment i from point P is $(x^2 + y'^2)^{1/2}$.
Solve: The electric field created by segment i at point P is

$$\vec{E}_i = \frac{\Delta q}{4\pi\varepsilon_0 (x^2 + y'^2)}(\cos\theta\hat{i} - \sin\theta\hat{j}) = \frac{\Delta q}{4\pi\varepsilon_0 (x^2 + y'^2)}\left(\frac{x}{\sqrt{x^2 + y'^2}}\hat{i} - \frac{y'}{\sqrt{x^2 + y'^2}}\hat{j}\right)$$

The net field is the sum of all the $\vec{E}_i$, which gives $\vec{E} = \sum_i \vec{E}_i$. Δq is not a coordinate, so before converting the sum to an integral we must relate charge Δq to length $\Delta y'$. This is done through the linear charge density $\lambda = Q/L$, from which we have the relationship

$$\Delta q = \lambda \Delta y' = \frac{Q}{L}\Delta y'$$

With this charge, the sum becomes

$$\vec{E} = \frac{Q/L}{4\pi\varepsilon_0}\sum_i \left[\frac{x\Delta y'}{(x^2 + y'^2)^{3/2}}\hat{i} - \frac{y'\Delta y'}{(x^2 + y'^2)^{3/2}}\hat{j}\right]$$

Now we let $\Delta y' \to dy'$ and replace the sum by an integral from $y' = 0$ m to $y' = L$. Thus,

$$\vec{E} = \frac{(Q/L)}{4\pi\varepsilon_0}\left(\int_0^L \frac{x\,dy'}{(x^2 + y'^2)^{3/2}}\hat{i} - \int_0^L \frac{y'\,dy'}{(x^2 + y'^2)^{3/2}}\hat{j}\right) = \frac{(Q/L)}{4\pi\varepsilon_0}\left(x\left[\frac{y'}{x^2\sqrt{x^2 + y'^2}}\right]_0^L \hat{i} - \left[\frac{-1}{\sqrt{x^2 + y'^2}}\right]_0^L \hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{1}{x\sqrt{x^2 + L^2}}\hat{i} - \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{Lx}\right)\left(1 - \frac{x}{\sqrt{x^2 + L^2}}\right)\hat{j}$$

23.45. Model: Assume that the ring of charge is thin and that the charge lies along circle of radius R.
Solve: From Example 23.4, the on-axis field of a ring of charge Q and radius R is

$$E_z = \frac{zQ}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}$$

When $z \ll R$, this means we are near the center of the ring. At that point, segments of charge i and j that are 180° apart create fields $\vec{E}_i$ and $\vec{E}_j$ that cancel each other. When the fields of all segments of charge around the ring are added, the net result is zero. This is indicated by the above expression because when $z = 0$ m, the electric field is zero. When $z \gg R$, then

$$(z^2 + R^2)^{3/2} = (z^2)^{3/2} = z^3$$

If this is used in the expression for E_z, we get

$$E_z \approx \frac{zQ}{4\pi\varepsilon_0 z^3} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}$$

This is the field of a point charge Q as seen along the z-axis.

23.46. Model: Assume that the ring of charge is thin and that the charge lies along circle of radius R.
Solve: (a) From Example 23.4, the on-axis field of a ring of charge Q and radius R is

$$E_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

For the field to be maximum at a particular value of z, $dE/dz = 0$. Taking the derivative,

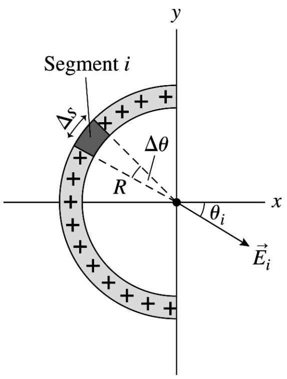
$$\frac{dE}{dz} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(z^2+R^2)^{3/2}} - \frac{z(3/2)(2z)}{(z^2+R^2)^{5/2}}\right] = 0 \Rightarrow \frac{1}{(z^2+R^2)^{3/2}} = \frac{3z^2}{(z^2+R^2)^{5/2}}$$

$$\Rightarrow 1 = \frac{3z^2}{z^2+R^2} \Rightarrow z = \pm\frac{R}{\sqrt{2}}$$

(b) The field strength at the point $z = R/\sqrt{2}$ is

$$(E_z)_{\max} = \frac{Q}{4\pi\varepsilon_0}\frac{(R/\sqrt{2})}{\left[(R/\sqrt{2})^2+R^2\right]^{3/2}} = \frac{2}{3\sqrt{3}}\frac{Q}{4\pi\varepsilon_0 R^2}$$

23.47. Model: Assume that the semicircular rod is thin and that the charge lies along the semicircle of radius R.
Visualize:



The origin of the coordinate system is at the center of the circle. Divide the rod into many small segments of charge Δq and arc length Δs. Segment i creates a small electric field $\vec{E}_i$ at the origin. The line from the origin to segment i makes an angle θ with the x-axis.

Solve: Because every segment i at an angle θ above the axis is matched by segment j at angle θ below the axis, the y-components of the electric fields will cancel when the field is summed over all segments. This leads to a net field pointing to the right with

$$E_x = \sum_i (E_i)_x = \sum_i E_i \cos\theta_i \qquad E_y = 0 \text{ N/C}$$

Note that angle θ_i depends on the location of segment i. Now all segments are at the same distance $r_i = R$ from the origin, so

$$E_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i^2} = \frac{\Delta q}{4\pi\varepsilon_0 R^2}$$

The linear charge density on the rod is $\lambda = Q/L$, where L is the rod's length. This allows us to relate charge Δq to the arc length Δs through

$$\Delta q = \lambda\Delta s = (Q/L)\Delta s$$

Thus, the net field at the origin is

$$E_x = \sum_i \frac{(Q/L)\Delta s}{4\pi\varepsilon_0 R^2}\cos\theta_i = \frac{Q}{4\pi\varepsilon_0 L R^2}\sum_i \cos\theta_i \Delta s$$

The sum is over all the segments on the rim of a semicircle, so it will be easier to use polar coordinates and integrate over θ rather than do a two-dimensional integral in x and y. We note that the arc length Δs is related to the small angle $\Delta\theta$ by $\Delta s = R\Delta\theta$, so

$$E_x = \frac{Q}{4\pi\varepsilon_0 L R}\sum_i \cos\theta_i \Delta\theta$$

With $\Delta\theta \to d\theta$, the sum becomes an integral over all angles forming the rod. θ varies from $\Delta\theta = -\pi/2$ to $\theta = +\pi/2$. So we finally arrive at

$$E_x = \frac{Q}{4\pi\varepsilon_0 L R}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{Q}{4\pi\varepsilon_0 L R}\sin\theta\Big|_{-\pi/2}^{\pi/2} = \frac{2Q}{4\pi\varepsilon_0 L R}$$

Since we're given the rod's length L and not its radius R, it will be convenient to let $R = L/\pi$. So our final expression for $\vec{E}$, now including the vector information, is

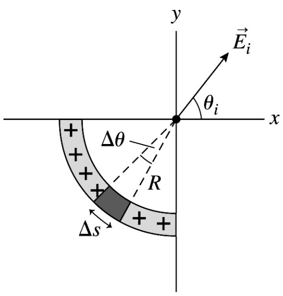
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\pi Q}{L^2}\right)\hat{i}$$

(b) Substituting into the above expression,

$$E = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)2\pi(30\times10^{-9}\text{ C})}{(0.10\text{ m})^2} = 1.70\times10^5 \text{ N/C}$$

23.48. Model: Assume that the quarter-circle plastic rod is thin and that the charge lies along the quarter-circle of radius R.

Visualize:



The origin of the coordinate system is at the center of the circle. Divide the rod into many small segments of charge Δq and arc length Δs.

Solve: (a) Segment i creates a small electric field $\vec{E}_i$ at the origin with two components:

$$(E_i)_x = E_i\cos\theta_i \qquad (E_i)_y = E_i\sin\theta_i$$

Note that the angle θ_i depends on the location of the segment i. Now all segments are at distance $r_i = R$ from the origin, so

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r_i^2} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}$$

The linear charge density of the rod is $\lambda = Q/L$, where L is the rod's length (L = quarter-circumference = $\pi R/2$). This allows us to relate charge Δq to the arc length Δs through

$$\Delta q = \lambda \Delta s = \left(\frac{Q}{L}\right)\Delta s = \left(\frac{2Q}{\pi R}\right)\Delta s$$

Using $\Delta s = R\Delta\theta$, the components of the electric field at the origin are

$$(E_i)_x = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\cos\theta_i = \frac{1}{4\pi_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\cos\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\cos\theta_i$$

$$(E_i)_y = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\sin\theta_i$$

(b) The x- and y-components of the electric field for the entire rod are the integrals of the expressions in part **(a)** from $\theta = 0$ rad to $\theta = \pi/2$. We have

$$E_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\cos\theta\, d\theta \qquad E_y = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\sin\theta\, d\theta$$

(c) The integrals are

$$\int_0^{\pi/2}\sin\theta\, d\theta = \left[-\cos\theta\right]_0^{\pi/2} = -\left(\cos\frac{\pi}{2} - \cos 0\right) = +1 \qquad \int_0^{\pi/2}\cos\theta\, d\theta = \left[\sin\theta\right]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = +1$$

The electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\pi R^2}(\hat{i} + \hat{j})$$

23.49. Model: The electric field obeys the principle of superposition. Model the hole (which has no charge) as a disk of positive charge (part of the infinite plate) superimposed with a disk of negative charge.

Visualize: We'll use the on-axis field of a disk of charge; label that axis as the z-axis. We are given $\eta = 3.2\ \mu\text{C/m}^2$, $R = 0.10$ m, and $z = 0.12$ m.

Solve: We'll get the total field from adding the field of the infinite plane to the field of a *negative* disk where the hole is.

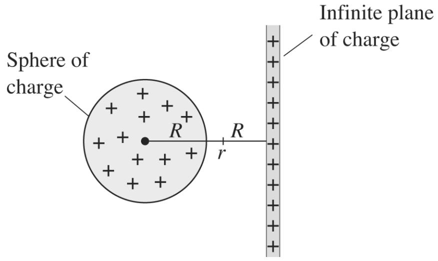
$$E_{\text{tot}} = E_{\text{plane}} + E_{\text{disk}} = \frac{\eta}{2\varepsilon_0} - \frac{\eta}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right] = \frac{\eta}{2\varepsilon_0}\left[\frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$= \frac{(3.2\ \mu\text{C/m}^2)}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[\frac{0.12\ \text{m}}{\sqrt{(0.12\ \text{m})^2 + (0.10\ \text{m})^2}}\right] = 1.4\times10^5\ \text{N/C}$$

Assess: The answer is in the ballpark of the answer for the example in the book. We also note that the answer depends linearly on η, which we expect. In the limit as $z \to \infty$ the hole becomes negligible and the field is as if it were due to an infinite plane of charge.

23.50. Model: The electric field of a sphere of charge is the same as if all the charge were concentrated at the center.

Visualize: Put the origin at the center of the sphere.



Solve: First express the charge on the sphere Q in terms of the given η.

$$\eta = \frac{Q}{A} = \frac{Q}{4\pi R^2} \Rightarrow Q = 4\pi R^2 \eta$$

In the figure the field from the plane points in the negative direction.

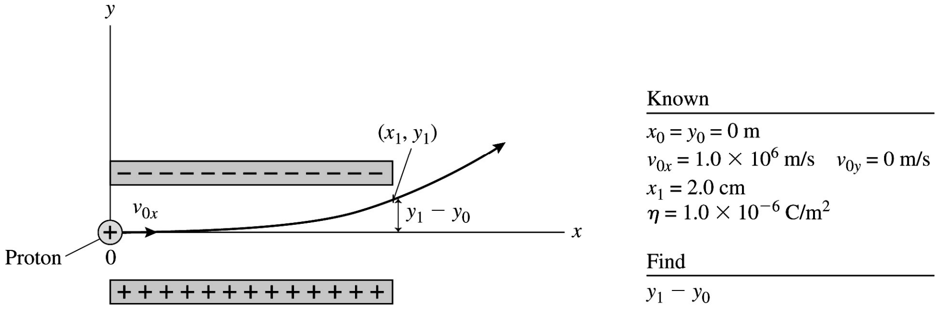
$$E_{\text{sphere}} + E_{\text{plane}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} - \frac{\eta}{2\varepsilon_0} = 0 \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\eta}{r^2} = \frac{\eta}{2\varepsilon_0} \Rightarrow \frac{R^2}{r^2} = \frac{1}{2} \Rightarrow r = \sqrt{2}R$$

This is the distance from the center of the sphere; the distance from the plane is $2R - \sqrt{2}R = (2 - \sqrt{2})R.$

Assess: The answer is outside the sphere (which is good). The charge density cancelled out, so this result stands regardless of the value of η.

23.51. Model: The electric field is uniform inside the capacitor, so constant-acceleration kinematic equations apply to the motion of the proton.

Visualize:



Solve: From Equation 23.31 and $Q/A = \eta$, the electric field between the parallel plates $\vec{E} = (\eta/\varepsilon_0)\hat{j}$. The force on the proton is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m} = \frac{q\eta}{m\varepsilon_0}\hat{j} \Rightarrow a_y = \frac{q\eta}{m\varepsilon_0}$$

Using the kinematic equation $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$\Delta y = y_1 - y_0 = (0\text{ m/s})(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0) = \frac{1}{2}\left(\frac{q\eta}{m\varepsilon_0}\right)(t_1 - t_0)^2$$

To determine $t_1 - t_0$, we consider the horizontal motion of the proton. The proton travels a distance of 2.0 cm at a constant speed of 1.0×10^6 m/s. The velocity is constant because the only force acting on the proton is due to the field between the plate along the y-direction. Using the same kinematic equation,

$$\Delta x = 2.0\times10^{-2}\text{ m} = v_{0x}(t_1 - t_0) + 0\text{ m} \Rightarrow (t_1 - t_0) = \frac{2.0\times10^{-2}\text{ m}}{1.0\times10^{6}\text{ m/s}} = 2.0\times10^{-8}\text{ s}$$

$$\Rightarrow \Delta y = \frac{1}{2}\frac{(1.60\times10^{-19}\text{ C})(1.0\times10^{-6}\text{ C/m}^2)(2.0\times10^{-8}\text{ s})^2}{(1.67\times10^{-27}\text{ kg})(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)} = 2.2\text{ mm}$$

23.52. Model: Assume that the electric field inside the capacitor is constant, so constant-acceleration kinematic equations apply.

Solve: (a) The force on the electron inside the capacitor is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

Because $\vec{E}$ is directed upward (from the positive plate to the negative plate) and $q = -1.60\times10^{-19}$ C, the acceleration of the electron is downward. We can therefore write the above equation as simply $a_y = qE/m.$ To determine *E*, we must first find a_y. From kinematics,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow 0.040\text{ m} = 0\text{ m} + v_0\cos45°(t_1 - t_0) + 0\text{ m}$$

$$\Rightarrow (t_1 - t_0) = \frac{(0.040\text{ m})}{(5.0\times10^6\text{ m/s})\cos45°} = 1.1314\times10^{-8}\text{ s}$$

Using the kinematic equations for the motion in the y-direction,

$$v_{1y} = v_{0y} + a_y\left(\frac{t_1 - t_0}{2}\right) \Rightarrow 0\text{ m/s} = v_0\sin45° + \left(\frac{qE}{m}\right)\left(\frac{t_1 - t_0}{2}\right)$$

$$\Rightarrow E = -\frac{2\text{ m }v_0\sin45°}{q(t_1 - t_0)} = -\frac{2(9.1\times10^{-31}\text{ kg})(5.0\times10^6\text{ m/s})\sin45°}{(-1.60\times10^{-19}\text{ C})(1.1314\times10^{-8}\text{ s})} = 3550\text{ N/C} = 3.6\times10^3\text{ N/C}$$

(b) To determine the separation between the two plates, we note that $y_0 = 0$ m and $v_{0y} = (5.0\times10^6$ m/s)sin 45°, but at $y = y_1$, the electron's highest point, $v_{1y} = 0$ m/s. From kinematics,

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) \Rightarrow 0\text{ m}^2/\text{s}^2 = v_0^2\sin^2 45° + 2a_y(y_1 - y_0)$$

$$\Rightarrow (y_1 - y_0) = -\frac{v_0^2\sin^2 45°}{2a_y} = -\frac{v_0^2}{4a_y}$$

From part **(a)**,

$$a_y = \frac{qE}{m} = \frac{(-1.60\times10^{-19}\text{ C})(3550\text{ N/C})}{9.1\times10^{-31}\text{ kg}} = -6.242\times10^{14}\text{ m/s}^2$$

$$\Rightarrow y_1 - y_0 = -\frac{(5.0\times10^6\text{ m/s})^2}{4(-6.242\times10^{14}\text{ m/s}^2)} = 0.010\text{ m} = 1.0\text{ cm}$$

This is the height of the electron's trajectory, so the minimum spacing is 1.0 cm.

23.53. Model: The electric field is uniform inside the capacitor, so constant-acceleration kinematic equations apply to the motion of the electron.

Visualize: The condition for the electron to not hit the negative plate is that its vertical velocity should just become zero as the electron reaches the plate.

Solve: The force on the electron inside the capacitor is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

$$\Rightarrow a_y = \frac{-(1.60\times10^{-19}\text{ C})(1.0\times10^4\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = -1.756\times10^{15}\text{ m/s}^2$$

The initial velocity v_0 has two components: $v_{0x} = v_0\cos45°$ and $v_{0y} = v_0\sin45°$. Because the electric field inside the capacitor is along the +y-direction, the electron has a negative acceleration that reduces the vertical velocity. We require $v_{1y} = 0$ m/s if it is not to hit the plate. Using kinematics,

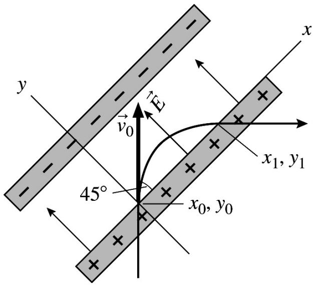
$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) \Rightarrow (0\text{ m/s})^2 = v_{0y}^2 + 2a_y\Delta y$$

$$\Rightarrow v_{0y} = \sqrt{-2a_y\Delta y} = \sqrt{-2(-1.756\times10^{15}\text{ m/s}^2)(0.02\text{ m})} = 8.381\times10^6\text{ m/s}$$

$$\Rightarrow v_0 = \frac{8.381\times10^6\text{ m/s}}{\sin45°} = 1.19\times10^7\text{ m/s}$$

23.54. Model: The parallel plates form a parallel-plate capacitor. The electric field inside a parallel-plate capacitor is a uniform field, so the electrons will have a constant acceleration.

Visualize:



Solve: (a) The bottom plate should be positive. The electron needs to be repelled by the top plate, so the top plate must be negative and the bottom plate positive. In other words, the electric field needs to point away from the bottom plate so the electron's acceleration $\vec{a}$ is toward the bottom plate.

(b) Choose an xy-coordinate system with the x-axis parallel to the bottom plate and the origin at the point of entry. Then the electron's acceleration, which is parallel to the electric field, is $\vec{a} = a\hat{j}$. Consequently, the problem looks just like a projectile problem. The kinetic energy $K = \frac{1}{2}mv_0^2 = 3.0\times10^{-17}$ J gives an initial speed $v_0 = (2K/m)^{1/2} = 8.115\times10^6$ m/s. Thus the initial components of the velocity are

$$v_{x0} = v_0\cos45° = 5.74\times10^6 \text{ m/s} \qquad v_{y0} = v_0\sin45° = 5.74\times10^6 \text{ m/s}$$

What acceleration $\vec{a}$ will cause the electron to pass through the point $(x_1, y_1) = (1.0\text{ cm}, 0\text{ cm})$? The kinematic equations of motion are

$$x_1 = x_0 + v_{x0}t_1 + \frac{1}{2}a_x t_1^2 = v_{x0}t_1 = 0.010\text{ m} \qquad y_1 = y_0 + v_{y0}t_1 + \frac{1}{2}a_y t_1 = v_{y0}t_1 + \frac{1}{2}at_1^2 = 0\text{ m}$$

From the x-equation, $t_1 = x_1/v_{x0} = 1.742\times10^{-9}$ s. Using this in the y-equation gives

$$a = -\frac{2v_{y0}t_1}{t_1^2} = -6.59\times10^{15}\text{ m/s}^2$$

But the acceleration of an electron in an electric field is

$$a = \frac{F_{\text{elec}}}{m} = \frac{q_{\text{elec}}E}{m} = \frac{-eE}{m} \Rightarrow E = -\frac{ma}{e} = -\frac{(9.11\times10^{-31}\text{ kg})(-6.59\times10^{15}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 37{,}500\text{ N/C} = 3.8\times10^4\text{ N/C}$$

(c) The minimum separation d_{min} must equal the "height" y_{max} of the electron's trajectory above the bottom plate. (If d were less than y_{max}, the electron would collide with the upper plate.) Maximum height occurs at $t = \frac{1}{2}t_1 = 8.71\times10^{-10}$ s. At this instant,

$$y_{\text{max}} = v_{y0}t + \frac{1}{2}at^2 = 0.0025\text{ m} = 2.5\text{ mm}$$

Thus, $d_{\text{min}} = 2.5$ mm.

23.55. Model: The orbital motion of the positron-electron system about their center of mass is due to the electric force between the positron and the electron.

Solve: The distance between the charges is twice the radius of their orbits about the center of mass. The force that causes the circular motion is

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_{\text{positron}}|q_{\text{electron}}|}{(2r^2)} = \frac{m_{\text{electron}}v^2}{r} = \frac{m_{\text{positron}}v^2}{r} = \frac{m_{\text{electron}}4\pi^2 f^2 r^2}{r}$$

where we used $v = 2\pi r f$. The frequency is

$$f = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_{\text{positron}}\left|q_{\text{electron}}\right|}{m_{\text{electron}}16\pi^2 r^3}} = \sqrt{\frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(9.1\times10^{-31}\ \text{kg})16\pi^2(0.50\times10^{-9}\ \text{m})^3}} = 1.13\times10^{14}\ \text{Hz}$$

23.56. Model: The long charged wire is an infinite line of charge. The charges on the wire and the plastic rod are uniform.
Visualize: The plastic stirrer is located on the x-axis.
Solve: The electric field of the infinite line of charge at a distance x from its axis is

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{x}$$

Because the electric field is a function of x, the plastic stirrer experiences an electric field that varies along the length of the stirrer. We handled such problems earlier. Take a small segment of the charge on the stirrer and calculate the electric force due to the line charge on this charge segment Δq. A summation of all such forces on the charge segments Δq will yield the net force on the stirrer. This procedure is equivalent to integrating the electric force on a small segment Δq over the length of the stirrer. The force on charge Δq of length Δx at position x due to the infinite line of charge is

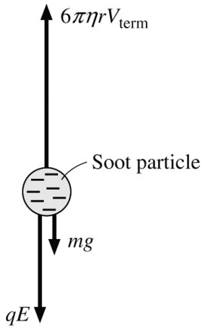
$$\Delta F = E\Delta q = E\lambda'\Delta x = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\lambda\lambda'\Delta x}{x}\right)$$

In the above expression, $\lambda' = Q/L$ is the linear charge density of the stirrer and λ is the linear charge density of the plastic rod. We have also used the relation $\Delta q/\Delta x = \lambda'$. Changing $\Delta x \to dx$ and integrating x from $x = 0.020$ m to $x = 0.080$ m, the total force on the stirrer is

$$F = \int_{0.020\ \text{m}}^{0.080\ \text{m}} \frac{1}{4\pi\varepsilon_0}\frac{2\lambda\lambda'}{x}dx = \left(\frac{1}{4\pi\varepsilon_0}\right)(2\lambda\lambda')\ln x\Big|_{0.020\ \text{m}}^{0.080\ \text{m}} = \left(\frac{1}{4\pi\varepsilon_0}\right)(2\lambda\lambda')\ln\left(\frac{0.080\ \text{m}}{0.020\ \text{m}}\right)$$

$$= (9.0\times10^9\ \text{N m}^2/\text{C}^2)2(1.0\times10^{-7}\ \text{C/m})\left(\frac{10\times10^{-9}\ \text{C}}{0.060\ \text{m}}\right)\ln(4) = 4.2\times10^{-4}\ \text{N}$$

23.57. Model: Assume the soot particle is a point charge in $F = qE$ and a small sphere where the drag force is concerned. Also assume the particle becomes negatively charged.
Visualize: Draw a free body diagram with the electric force directed down because the E field is down, but the actual direction of that force depends on the sign of q.



Solve: **(a)** Write Newton's first law using the free body diagram.

$$\Sigma F = 6\pi\eta r v_{\text{term}} - mg - qE = 0$$

Note that $m = V\rho = \frac{4}{3}\pi r^3 \rho$. Solve for v_{term}.

$$v_{\text{term}} = \frac{\left(\frac{4}{3}\pi r^3 \rho\right)g + qE}{6\pi\eta r}$$

(b) With the qE term equal to zero the π and an r will cancel.

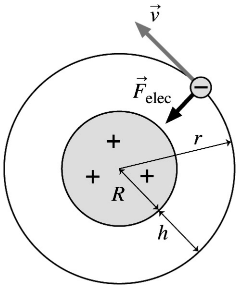
$$v_{\text{term}} = \frac{\frac{4}{3}r^2\rho g}{6\eta} = \frac{\frac{4}{3}(0.50\ \mu\text{m})^2(2200\ \text{kg/m}^3)(9.8\ \text{m/s}^2)}{6(1.8\times10^{-5}\ \text{kg/m}\cdot\text{s})} = 0.067\ \text{mm/s}$$

(c) The charge on the soot particle is negative, so the electric force is up.

$$v_{\text{term}} = \frac{\frac{4}{3}\pi(0.50\ \mu\text{m})^3(2200\ \text{kg/m}^3)(9.8\ \text{m/s}^2) + (125)(-1.60\times10^{-19}\ \text{C})(150\ \text{N/C})}{6\pi(1.8\times10^{-5}\ \text{kg/m}\cdot\text{s})(0.50\ \mu\text{m})} = 0.049\ \text{mm/s}$$

Assess: The terminal speed is smaller when the electric force is present, as we would expect.

23.58. Model: An orbiting electron experiences a force that causes centripetal acceleration.
Visualize:



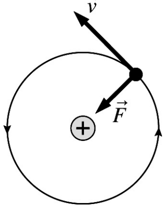
Solve: The electron orbits at a radius $r = R + h = \frac{1}{2}(2.0\ \text{mm}) + 1.0\ \text{mm} = 2.0\ \text{mm}$. The force that causes the circular motion is simply the electric force $\vec{F}$ given by Coulomb's law:

$$F = \frac{K|Q_{\text{sphere}}||q_{\text{elec}}|}{r^2} = \frac{KeQ_{\text{sphere}}}{r^2} = \frac{(9\times10^9\ \text{N m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})(1.0\times10^{-9}\ \text{C})}{(2.0\times10^{-3}\ \text{m})^2} = 3.60\times10^{-13}\ \text{N}$$

For circular motion,

$$F = ma_{\text{centripetal}} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{rF}{m}} = \sqrt{\frac{(2.0\times10^{-3}\ \text{m})(3.6\times10^{-13}\ \text{N})}{9.11\times10^{-31}\ \text{kg}}} = 2.8\times10^7\ \text{m/s}$$

23.59. Model: The electron orbiting the proton experiences a force given by Coulomb's law.
Visualize:



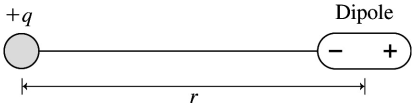
Solve: The force that causes the circular motion is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{p}}|q_{\text{e}}|}{r^2} = \frac{m_{\text{e}}v^2}{r} = \frac{m_{\text{e}}(4\pi^2 r^2 f^2)}{r}$$

where we used $v = 2\pi r/T = 2\pi rf$. The frequency is

$$f = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_\text{p}|q_\text{e}|}{4\pi^2 m_\text{e} r^3}} = \sqrt{\frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{4\pi^2(9.11\times10^{-31}\text{ kg})(5.3\times10^{-11}\text{ m})^3}} = 6.56\times10^{15}\text{ Hz}$$

23.60. Model: The electric field at the dipole's location is that of the ion with charge q.
Visualize:



Solve: **(a)** We have $p = \alpha E$. The units of α are the units of p/E and are

$$\frac{\text{C m}}{\text{N/C}} = \frac{\text{C}^2\text{ m}}{\text{N}} = \frac{\text{C}^2\text{s}^2}{\text{kg}}$$

(b) The electric field due to the ion at the location of the dipole is

$$E_\text{at dipole} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}, \text{ away from } q\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{i}$$

Because $\vec{p} = \alpha\vec{E}$, the induced dipole moment is

$$\vec{p} = \alpha\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\hat{i}$$

From Equation 23.10, the electric field produced by the dipole at the location of the ion is

$$\vec{E}_\text{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2}{r^3}\right)\alpha\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\hat{i} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{2q\alpha}{r^5}\right)\hat{i}$$

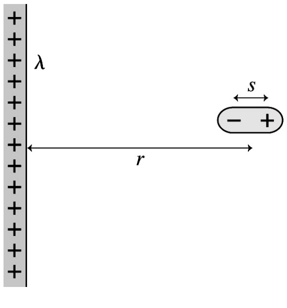
The force the dipole exerts on the ion is

$$\vec{F}_\text{dipole on ion} = q\vec{E}_\text{dipole} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{2q^2\alpha}{r^5}\right)\hat{i}$$

According to Newton's third law, $\vec{F}_\text{dipole on ion} = -\vec{F}_\text{ion on dipole}$. Therefore,

$$\vec{F}_\text{ion on dipole} = \left(\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{2q^2\alpha}{r^5}, \text{ toward ion}\right)$$

23.61. Model: The electric field at the dipole's location is that of the infinite line of charge of linear charge density λ.
Visualize:



Solve: The field at distance r from an infinite line of charge is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\hat{r}$$

It points straight away from the line. The dipole consists of charge $+q$ at distance $r + s/2$ and charge $-q$ at distance $r - s/2$. The net force on the dipole due to the field of the line is

$$\vec{F} = q\vec{E}_+ - q\vec{E}_- = \frac{2q\lambda}{4\pi\varepsilon_0}\left(\frac{1}{r+s/2} - \frac{1}{r-s/2}\right)\hat{r} = \frac{2q\lambda}{4\pi\varepsilon_0}\left(\frac{(r-s/2)-(r+s/2)}{(r+s/2)(r-s/2)}\right)\hat{r}$$

$$= -\frac{2qs\lambda}{4\pi\varepsilon_0(r^2-(s/2)^2)}\hat{r} = -\frac{2p\lambda}{4\pi\varepsilon_0(r^2-(s/2)^2)}\hat{r}$$

where we used $p = qs$ for the dipole moment. The minus sign shows that this is an attractive force, toward the line of charge. If $r >> s$, the $(s/2)^2$ term in the denominator is negligible and we find that the line of charge exerts an attractive force of magnitude

$$F = \frac{2p\lambda}{4\pi\varepsilon_0 r^2}$$

23.62. Model: Model the ozone molecule as a uniform rod.

Visualize: When the molecule is rotated there is a restoring torque tending to return it to equilibrium. This has the form of a Hooke's law if the angle is small. The rod-like molecule will vibrate around an axis through its center, so $I = \frac{1}{12}ML^2$.

Solve: The torque is in the opposite direction from the angle of rotation: $\tau = -pE\sin\theta$. We combine this with the rotational form of Newton's second law: $\tau = I\alpha$.

$$\alpha = \frac{-pE\sin\theta}{I}$$

But α is the second derivative of θ.

$$\frac{d^2\theta}{dt^2} = \frac{-pE\sin\theta}{I}$$

Because the angle is small we can approximate $\sin\theta \approx \theta$.

$$\frac{d^2\theta}{dt^2} = \frac{-pE}{I}\theta$$

From our chapter on simple harmonic motion we know that the general form of the differential equation is

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Match this up with what we have and see that the angular frequency of the oscillation is

$$\omega = \sqrt{\frac{pE}{I}} = \sqrt{\frac{pE}{\frac{1}{12}ML^2}} = \sqrt{\frac{(1.8\times10^{-30}\ \text{C}\cdot\text{m})(5000\ \text{N/C})}{\frac{1}{12}(3\cdot16\ \text{u}\cdot1.66\times10^{-27}\ \text{kg/u})(2.5\times10^{-10}\ \text{m})^2}} = 4.657\ \text{GHz}$$

The frequency f is then $f = \omega/2\pi = (4.657\ \text{GHz})/2\pi = 0.74\ \text{GHz}$.

Assess: The tiny molecule oscillates back and forth many, many times per second.

23.63. Solve: (a) Charges ±2.0 nC form an electric dipole. The electric field strength 2.5 cm from the dipole in the plane perpendicular to the dipole is 1150 N/C. What is the charge separation?

(b) Solving for s,

$$s = \frac{(1150\ \text{N/C})(0.025\ \text{m})^3}{(9\times10^9\ \text{N m}^2/\text{C}^2)(2.0\times10^{-9}\ \text{C})} = 9.98\times10^{-4}\ \text{m} = 1.0\times10^{-3}\ \text{m} = 1.0\ \text{mm}$$

23.64. Solve: (a) A very long charged wire has linear charge density 2.0×10^{-7} C/m. At what distance from the wire is the field strength 25,000 N/C?
(b) Solving for r,

$$r=\frac{(9\times10^{9}\text{ N m}^2/\text{C}^2)2(2.0\times10^{-7}\text{ C/m})}{25{,}000\text{ N/C}}=0.144\text{ m}\approx14\text{ cm}$$

23.65. Solve: (a) At what distance along the axis of a charged disk is the electric field strength half its value at the center of the disk? Give your answer in terms of the disk's radius R.
(b) Dividing both sides of the equation by $\eta/2\varepsilon_0$,

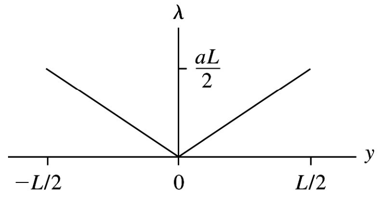
$$1-\frac{z}{\sqrt{z^2+R^2}}=\frac{1}{2}\Rightarrow 2z=\sqrt{z^2+R^2}\Rightarrow 4z^2=z^2+R^2\Rightarrow z=\frac{R}{\sqrt{3}}$$

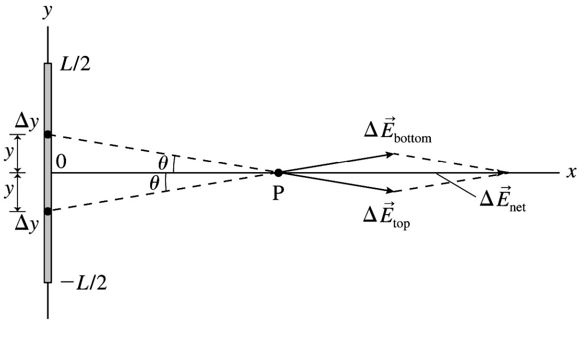
23.66. Solve: (a) A proton is released from the positive plate of a parallel-plate capacitor and accelerates toward the negative plate at $2.0\times10^{12}\text{ m/s}^2$. If the capacitor plates are $2.0\text{ cm}\times2.0\text{ cm}$ squares, what is the magnitude of the charge on each?
(b) Solve the first equation for E and substitute into the second equation. The charge is

$$Q=\frac{(1.67\times10^{-27}\text{ kg})(2.0\times10^{12}\text{ m/s}^2)(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)(0.020\text{ m})^2}{1.60\times10^{-19}\text{ C}}=7.4\times10^{-11}\text{ C}$$

Challenge Problems

23.67. Model: The rod is thin and is assumed to be a line of charge of length L.
Visualize:





y
$L/2$
Δy
y
0
θ
θ
y
Δy
P
$\Delta\vec{E}_{\text{bottom}}$
x
$\Delta\vec{E}_{\text{net}}$
$\Delta\vec{E}_{\text{top}}$
$-L/2$

Solve: (a) The λ-versus-y graph over the length of the rod is shown in the figure.
(b) Consider a segment of charge Δq of length Δy at a distance y from the center of the rod. The amount of charge in this segment is

$$\Delta q=\lambda\Delta y=a|y|\Delta y$$

Converting Δq to dq, Δy to dy, and integrating from $y=-L/2$ to $y=+L/2$, the total charge is

$$Q=\int dq=\int_{-L/2}^{+L/2}a|y|dy=2\int_{0}^{L/2}ay\,dy=2a\frac{y^2}{2}\bigg|_0^{L/2}=\frac{aL^2}{4}$$

Thus the constant a is

$$a=\frac{4Q}{L^2}$$

(c) With the origin of the coordinate system at the center of the rod, consider two symmetrically located charge segments Δq with length Δy. The electric field at P from the top charge segment makes an angle θ below the x-axis and the electric field of the bottom charge segment makes an angle θ above the x-axis. Because the charge density is symmetric about the origin (i.e., λ (at $-y$) = λ (at y)), the y-components of the two contributions cancel out. Thus, we have to calculate only the x-component of the electric field at P. Because the electric field strength of the lower half of the rod is the same as that of the upper half, we only need to obtain the electric field strength of half the rod, then multiply by two. The electric field along the x-direction due to a charge segment Δq is

$$\Delta E_x = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{(x^2+y^2)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\Delta y}{(x^2+y^2)}\frac{x}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{xa|y|\Delta y}{(x^2+y^2)^{3/2}}$$

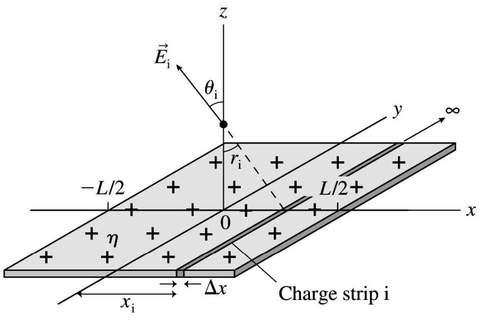
Changing ΔE to dE, $\Delta y = dy$, integrating y from $y = 0$ m to $y = L/2$, and multiplying by 2 to take into account the entire rod, the electric field is

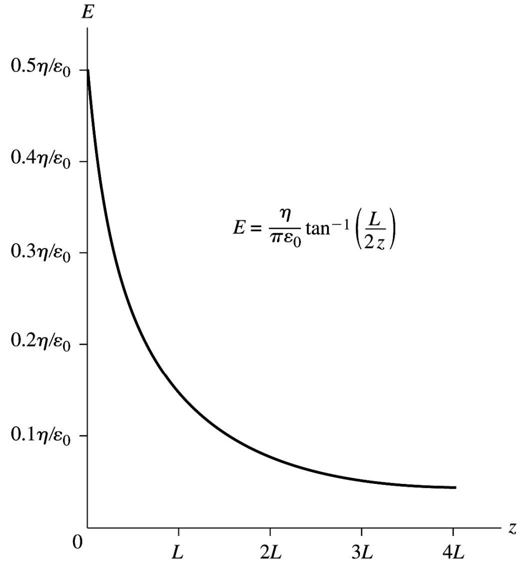
$$E_x = 2\int_0^{L/2}\frac{ax}{4\pi\varepsilon_0}\frac{y\,dy}{(x^2+y^2)^{3/2}} = \frac{2ax}{4\pi\varepsilon_0}\int_0^{L/2}\frac{y\,dy}{(x^2+y^2)^{3/2}} = \frac{2ax}{4\pi\varepsilon_0}\left[\frac{-1}{\sqrt{x^2+y^2}}\right]_0^{L/2}$$

$$= \frac{2ax}{4\pi\varepsilon_0}\left[\frac{1}{x}-\frac{1}{\sqrt{x^2+L^2/4}}\right] = \frac{8Q}{4\pi\varepsilon_0 L^2}\left[1-\frac{x}{\sqrt{x^2+L^2/4}}\right]$$

The last step used the expression for a from part **(b)**.

23.68. Model: Model the infinitely long sheet of charge with width L as a uniformly charged sheet.





Solve: **(a)** Divide the sheet into many long strips parallel to the y-axis and of width Δx. Each strip has a linear charge density $\lambda = \eta\Delta x$ and acts like a long, charged wire. At the point of interest, strip i contributes a small field $\vec{E}_i$ of strength

$$E_i = \frac{2\lambda}{4\pi\varepsilon_0 r_i} = \frac{2\eta\Delta x}{4\pi\varepsilon_0 r_i}$$

By symmetry, the x-components of all the strips will add to zero and the net field will point straight away from the sheet along the z-axis. The net field's z-component will be

$$E_z = \sum_i (E_i)_z = \sum_i E_i\cos\theta_i$$

The distance $r_i = (x_i^2+z^2)^{1/2}$ and $\cos\theta_i = z/r_i = z/(x_i^2+z^2)^{1/2}$. Thus,

$$E_z = \sum_i \frac{2\eta\Delta x}{4\pi\varepsilon_0(x_i^2+z^2)^{1/2}}\frac{z}{(x_i^2+z^2)^{1/2}} = \frac{2\eta z}{4\pi\varepsilon_0}\sum_i\frac{\Delta x}{x_i^2+z^2}$$

If we now let $\Delta x \to dx$, the sum becomes an integral ranging from $x = \Delta L/2$ to $x = L/2$. This gives

$$E_z = \frac{2\eta z}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dx}{x^2+z^2} = \frac{\eta z}{2\pi\varepsilon_0}\left[\frac{1}{z}\tan^{-1}\left(\frac{x}{z}\right)\right]_{-L/2}^{L/2} = \frac{\eta}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{L}{2z}\right) - \tan^{-1}\left(\frac{-L}{2z}\right)\right]$$

From trigonometry, $\tan^{-1}(-\phi) = -\tan^{-1}(\phi)$. So finally,

$$E_z = \frac{\eta}{\pi\varepsilon_0}\tan^{-1}\left(\frac{L}{2z}\right) \Rightarrow \vec{E} = \frac{\eta}{\pi\varepsilon_0}\tan^{-1}\left(\frac{L}{2z}\right)\hat{k}$$

(b) As $z \to 0$ m,

$$\tan^{-1}\left(\frac{L}{2z}\right) \to \frac{\pi}{2} \Rightarrow \vec{E} \to \frac{\eta}{\pi\varepsilon_0}\frac{\pi}{2}\hat{k} = \frac{\eta}{2\varepsilon_0}\hat{k}$$

This is the electric field due to a plane as you can see from Equation 23.28. We obtain this result because in the limit as $z \to 0$ m, the dimension L becomes extremely large. As $z \to \infty$,

$$\tan^{-1}\left(\frac{L}{2z}\right) \to \frac{L}{2z} \Rightarrow \vec{E} \to \frac{\eta}{\pi\varepsilon_0}\frac{L}{2z}\hat{k} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{k}$$

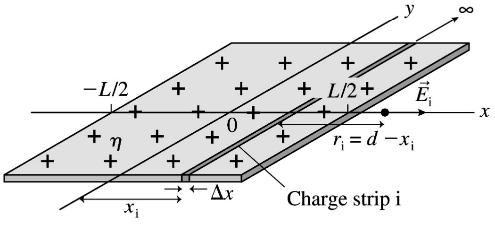
where we have used $\eta L = \lambda$ as the charge per unit length of the sheet. This is the electric field due to a long, charged wire. We obtain this result because for $z >> L$, the infinitely long sheet "looks" like an infinite line charge.

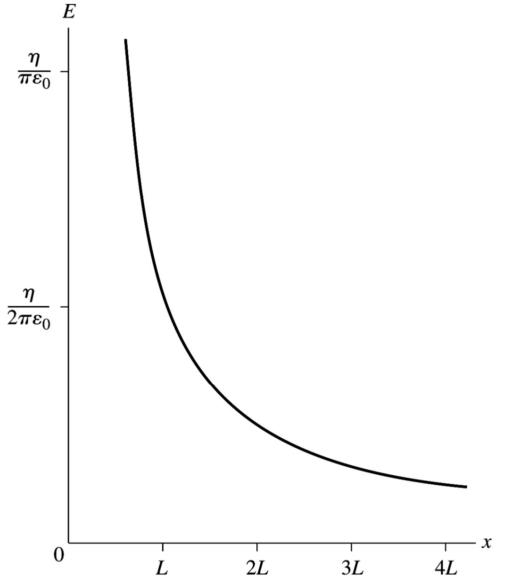
(c) The following table shows the field strength E_z in units of η/ε_0 for selected values of z in units of L. A graph of E_z is shown in the figure on the previous page.

z/L	$E_z\big/\frac{\eta}{\varepsilon_0}$
0	0.5
0.25	0.35
0.50	0.25
1.0	0.15
2.0	0.08
3.0	0.05
4.0	0.04

23.69. Model: Model the infinitely long sheet of charge with width L as a uniformly charged sheet.

Visualize:





Solve: (a) Consider a point on the x-axis at a distance d from the center of the sheet of charge. (We'll call this distance d to begin with, rather than x, to avoid confusion with x as the integration variable.) Once again, let the sheet of charge be divided into small strips of width Δx. Each strip has a linear charge density $\lambda = \eta\Delta x$ and acts like a long, charged wire. Strip i is at distance $r_i = d - x_i$ from the point of interest, so it contributes the small field

$$\vec{E}_i = \frac{2\lambda}{4\pi\varepsilon_0 r_i}\hat{i} = \frac{2\eta\Delta x}{4\pi\varepsilon_0 (d - x_i)}\hat{i}$$

In this situation all fields $\vec{E}_i$ point in the same direction. Their x-components all add to give a net field in the $+x$-direction:

$$E_x = \sum_i (E_i)_x = \frac{2\eta}{4\pi\varepsilon_0}\sum_i \frac{\Delta x}{(d - x_i)}$$

We'll replace the sum with an integral from $x = -L/2$ to $x = +L/2$, giving

$$E_x = \frac{2\eta}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dx}{(d-x)} = \frac{2\eta}{4\pi\varepsilon_0}\big[-\ln(d-x)\big]_{-L/2}^{L/2} = \frac{2\eta}{4\pi\varepsilon_0}\big[-\ln(d-L/2)+\ln(d+L/2)\big] = \frac{2\eta}{4\pi\varepsilon_0}\ln\left(\frac{2d+L}{2d-L}\right)$$

Now that the integration is complete, we can note that d really is the x-coordinate of the point of interest. Substituting x for d and changing to vector form, we end up with

$$\vec{E}_{\text{net}} = \frac{2\eta}{4\pi\varepsilon_0}\ln\left(\frac{2x+L}{2x-L}\right)\hat{i}$$

(b) If the point is very distant compared to width of the sheet of charge ($x >> L$), then the sheet of charge looks like a line of charge with linear density (charge per unit length) $\lambda = \eta L$. Write

$$E_x = \frac{2\eta}{4\pi\varepsilon_0}\ln\left(\frac{1+L/2x}{1-L/2x}\right) = \frac{2\eta}{4\pi\varepsilon_0}\big[\ln(1+L/2x) - \ln(1-L/2x)\big]$$

If $x >> L$, then $L/2x << 1$ and we can use the approximation $\ln(1+u) \approx u$ if $u << 1$. Thus

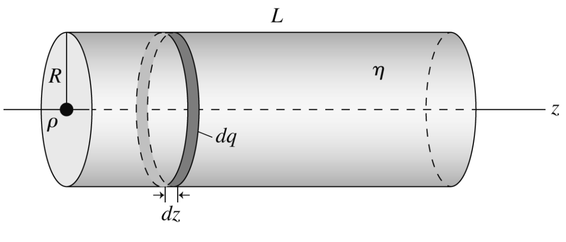
$$E_x \approx \frac{2\eta}{4\pi\varepsilon_0}\left[\frac{L}{2x} - \left(-\frac{L}{2x}\right)\right] = \frac{2\eta L}{4\pi\varepsilon_0 x} = \frac{2\lambda}{4\pi\varepsilon_0 x}$$

This is the field of a line of charge with $\lambda = \eta L$, as we expected.

(c) The following table shows the field strength E_x in units of $2\eta/4\pi\varepsilon_0$ for selected values of z in units of L. A graph of E_x is shown in the figure above.

z/L	$E_x \big/ \frac{2\eta}{4\pi\varepsilon_0}$
0.75	1.61
1.0	1.10
1.5	0.69
2.0	0.51
3.0	0.34
4.0	0.25

23.70. Model: Model the cylindrical shell as a stack of rings of charge
Visualize: We seek the field at point P.



Solve: First find the charge on the cylinder.

$$\eta = \frac{Q}{A} = \frac{Q}{2\pi RL} \Rightarrow Q = 2\pi RL\eta$$

The charge on one of the rings of width dz is then $dq = 2\pi R\eta dz$.

We will add up (with an integral) the contributions from all the rings. The on-axis electric field for a ring of charge is

$$E_{\text{ring}} = dE = \frac{1}{4\pi\varepsilon_0}\frac{z\,dq}{(z^2+R^2)^{3/2}}$$

$$E_{\text{cylinder}} = \int dE = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{z\,dq}{(z^2+R^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\int_0^L 2\pi R\eta \frac{z\,dz}{(z^2+R^2)^{3/2}} = \frac{R\eta}{2\varepsilon_0}\int_0^L \frac{z\,dz}{(z^2+R^2)^{3/2}}$$

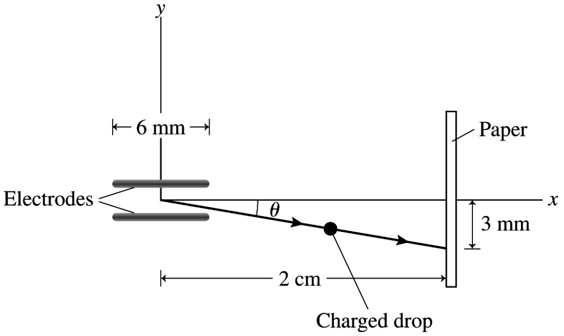
To do this integral, use a u-substitution: $u = z^2 + R^2 \Rightarrow du = 2zdz$.

$$E_{\text{cylinder}} = \frac{R\eta}{2\varepsilon_0}\int_0^L \frac{z\,dz}{(z^2+R^2)^{3/2}} = \frac{R\eta}{2\varepsilon_0}\left[\frac{-1}{\sqrt{z^2+R^2}}\right]_0^L = \frac{R\eta}{2\varepsilon_0}\left[\frac{-1}{\sqrt{L^2+R^2}} - \frac{-1}{R}\right] = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{R}{\sqrt{L^2+R^2}}\right]$$

Assess: In the limit as $R \to 0$ the field looks like that from an infinite plane. In the limit as $R \to \infty$ the field goes to zero.

23.71. Model: The two electrodes form a parallel-plate capacitor. The electric field inside the electrodes is uniform, so the electrons have constant acceleration.

Visualize:



Solve: The ink drops are deflected up or down as they experience an electric field between the two parallel electrodes. The electric field exerts a force on the ink drops which is

$$F = qE = ma_y \Rightarrow E = ma_y/q$$

We therefore need to determine the mass m, the charge q, and the acceleration a_y of the ink drops. We have

$$m = \rho V = \rho\left(\frac{4\pi}{3}r^3\right) = (800\text{ kg/m}^3)\left[\frac{4\pi}{3}(15\times10^{-6}\text{ m})^3\right] = 1.131\times10^{-11}\text{ kg}$$

$$q = (8\times10^5)(1.60\times10^{-19}\text{ C}) = 1.28\times10^{-13}\text{ C}$$

At maximum deflection, the drop's angle upon exiting the plates must be

$$\tan\theta = \frac{v_y}{v_x} = \left(\frac{3\text{ mm}}{2.0\text{ cm}}\right) = \pm0.15 \Rightarrow v_y = (\pm0.15)v_x = (\pm0.15)(20\text{ m/s}) = \pm3.0\text{ m/s}$$

From the kinematic equation $v_{1y} = v_{0y} + a_y(t_1 - t_0)$,

$$a_y = \frac{v_{1y} - v_{0y}}{t_1 - t_0} \Rightarrow a_y = \frac{\pm3.0\text{ m/s} - 0\text{ m/s}}{t_1 - t_0}$$

We can obtain $t_1 - t_0$ from the x-motion between the plates as follows:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow x_1 - x_0 = 6.0\text{ mm} = v_{0x}(t_1 - t_0) + 0\text{ m}$$

$$\Rightarrow t_1 - t_0 = \frac{6.0\times10^{-3}\text{ m}}{v_{0x}} = \frac{6.0\times10^{-3}\text{ m}}{20\text{ m/s}} = 3.0\times10^{-4}\text{ s} \Rightarrow a_y = \frac{\pm3.0\text{ m/s}}{3.0\times10^{-4}\text{ s}} = \pm1.0\times10^4\text{ m/s}^2$$

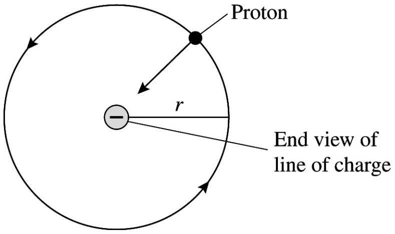
We are now in a position to obtain the field strength E from the equation

$$E = \frac{ma_y}{q} = \frac{(1.131\times10^{-11}\text{ kg})(1.0\times10^4\text{ m/s}^2)}{1.28\times10^{-13}\text{ C}} = 8.84\times10^5\text{ N/C}$$

The electric field strength is 8.8×10^5 N/C.

Assess: Because of the large acceleration due to the electric field, the acceleration due to gravity can be ignored in the calculations.

23.72. Model: The electric field is that of an infinite line charge.
Visualize:



Solve: The wire must be negative to attract the proton. From Equation 23.17, the electric field strength due to an infinite line of charge at a distance r from the line charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2|\lambda|}{r}$$

The force on the proton due to this electric field causes the centripetal acceleration:

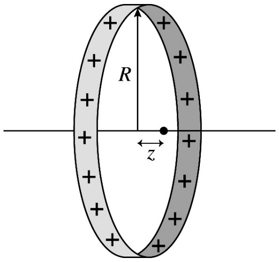
$$F = qE = \frac{1}{4\pi\varepsilon_0}\frac{2q|\lambda|}{r} = \frac{mv^2}{r} \Rightarrow |\lambda| = (4\pi\varepsilon_0)\frac{mv^2}{2q}$$

Using $v = 2\pi r/T = 2\pi rf$,

$$|\lambda| = (4\pi\varepsilon_0)\frac{m(4\pi^2r^2f^2)}{2q} = \frac{(1.67\times10^{-27}\text{ kg})4\pi^2(1.0\times10^{-2}\text{ m})^2(1.0\times10^6\text{ s}^{-1})^2}{(9.0\times10^9\text{ N m}^2/\text{C}^2)2(1.60\times10^{-19}\text{ C})} = 2.3\times10^{-9}\text{ C/m}$$

Because the wire has to be negative, $\lambda = -2.3$ nC/m.

23.73. Model: The electric field is that of a positively charged ring. The ring is thin and the charge lies along circle of radius R.
Visualize:



Solve: **(a)** From Example 23.4, the electric field on the axis of a ring of radius R is

$$E_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

The force on negative charge $-q$ is

$$F_z = -qE_z = -\frac{1}{4\pi_0}\frac{qQz}{(z^2+R^2)^{3/2}}$$

The meaning of the negative sign is that the force on the electron points left when the displacement is to the right and to the right when the displacement is to the left. That is, the electric force tries to keep the charge at $z = 0$ m.

(b) For small amplitude oscillations, that is, when $z << R$, the force is

$$F_z = -\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^3}z\left(1+\frac{z^2}{R^2}\right)^{-3/2} = -\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^3}z\left(1-\frac{3z^2}{2R^2}+\cdots\right) \approx -\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^3}z$$

This is simply Hooke's law $F_z = -kz$ where the "spring constant" is

$$k = \frac{qQ}{4\pi\varepsilon_0 R^3}$$

A particle that obeys Hooke's law undergoes simple harmonic motion with a frequency given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 m R^3}}$$

(c) Substituting into the above expression,

$$f = \frac{1}{2\pi}\sqrt{\frac{(1.0\times10^{-13}\text{ C})(1.6\times10^{-19}\text{ C})(9.0\times10^{9}\text{ N/m}^2\text{ C}^2)}{(9.11\times10^{-31}\text{ kg})(1.0\times10^{-6}\text{ m})^3}} = 2.0\times10^{12}\text{ Hz}$$